The Agency Costs of On-the-Job Search

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Abstract. This paper studies how on-the-job search (OJS) by a worker shapes the incentive structure and the optimal incentive contract in a long-term employment relationship modeled as a repeated principal-agent interaction. OJS generates a tradeoff between rent-extraction and efficiency, because the agent's incentives to conduct OJS depend on the rent he receives in the employment relationship. The optimal incentive contract for job seekers differs notably from that for non-searching agents in that it pays excessive bonuses as well as an efficiency wage. The results suggest a link between the concomitant rise in the levels of pay and the use of performance pay on the one side, and the increase in the incidence of OJS on the other. Further, when the parties can renegotiate the incentive contract, the employment relationship exhibits non-trivial wage and turnover dynamics that are highly consistent with empirical regularities.

Keywords: Repeated Agency, On-the-Job Search, Multitasking, Efficiency Wages, Excessive Bonuses.

JEL Classification Numbers: C73; D86; J31; J33; L14; M12

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1 Introduction

On-the-job search (OJS hereafter) is an inherent part of today's labor market. In many industries, a substantial number of workers are constantly looking for alternative employment opportunities.[1] Further, job search is a time-consuming and cognitively demanding activity that includes careful screening of job ads, writing applications, preparing for and participating in job interviews, traveling, and so forth. Mueller (2010) reports that on every day on which they search, employed job seekers devote more than 100 minutes on average to search activities. Similarly, Arellano and Meghir (1992) find that job search has a negative impact on hours worked. This evidence suggests that search activities compete with work activities for the time and the attention of employed job seekers.[2] One motive for conducting OJS is that it allows workers to find better jobs. Further, since locating better job matches improves the matching process in the labor market, OJS is socially valuable. At the same time, employers oftentimes match offers from competing firms to retain their employees (Barron, Berger, and Black (2006) and Yamaguchi (2010)). Hence, OJS also gives rise to rent seeking by helping workers to extract a higher share of the match surplus of their current employment relationship. Search activities that are conducted out of such rent-seeking considerations are socially wasteful and should be prevented. In turn, workers' incentives to engage in inefficient OJS will depend on their employers' willingness to share the rents from employment with them.

The preceding considerations suggest that knowing how worker's incentives to engage in OJS interact with the incentive structure in organizations is a pressing issue for many firms. This paper investigates the impact of OJS by an agent on the incentive problem and the shape of the optimal incentive contract in a long-term employment relationship modeled as a repeated principal-agent interaction. The agent's incentives to engage in wasteful OJS activities are decreasing in the rent that he receives in the relationship, implying that the principal faces a tradeoff between rent-extraction and efficiency. The optimal incentive contract for an employed job seeker differs notably from the one for a non-searching worker in that it pays an efficiency wage as well as excessive performance bonuses. As a consequence, the optimal contract induces the employee to overwork. Further, when allowing for renegotiation of the incentive contract, OJS gives rise to increasing wage-tenure and productivity-tenure profiles and to a separation rate that decreases with the worker's tenure.

The paper's results provide an alternative perspective on the escalation of incentive pay and pay levels that has occurred especially at the upper end of the income distribution over the last

[1]Examining data from the Current Population Survey, Fallick and Fleischman (2004) report that on average 4.4% of US employees are engaged in active OJS. Using the same data source, Stevenson (2009) reports that in 2003, 13.7% of US employees were looking for a job online. Further, using evidence from the 1983-84 Labour Force Survey, Pissarides and Wadsworth (1994) find that at any point in time, 5.3% of UK employees were actively looking for a job.

[2]Search activities will generally lower a worker's on-the-job productivity even when conducted during free time. First, an intensive job search leaves little time for productive activities such as working extra hours or recovering from a hard day's work. Further, the worker may alter his behavior at the workplace to maximize the returns from search. For instance, job seekers may focus on activities whose outcomes are more easily observed by outsiders at the expense of less salient but more productive tasks. Finally, the cognitive and psychological burden of job search can divert the worker's attention from tasks he is supposed to perform on the job.

twenty to thirty years (see Lazear and Shaw (2007) and Lemieux, MacLeod, and Parent (2009)). In particular, the results suggest that this development may at least in part reflect an optimal adjustment of firms' compensation policies to increased job search activities by their employees. The paper's implications are also consistent with the observation that both, incentive pay and bargaining power are higher among high-skill workers who are at the same time more likely to conduct OJS than less skilled workers.[3] Further, while increased labor market mobility has been widely lauded as improving match formation, this paper points to a potential downside of a more mobile workforce. Specifically, increased incentives to look for alternative jobs create agency costs and cause firms to offer incentive contracts that induce overworking.

The intuition behind the results just described is that a worker who faces strong incentives to conduct OJS is more difficult to motivate than a worker who does not search. Hence, employed job-seekers must face more high-powered incentives to exert a given level of effort than non-searching workers. Further, firms that pay their employees just slightly more than their outside options will promote OJS as workers have an additional incentive to search for other jobs, namely to improve their bargaining position. By contrast, employers that have built a reputation for paying generous wages irrespective of their employees' outside options will be more successful in preventing or at least limiting OJS.

I formalize these ideas in an infinitely repeated principal-agent model in which in every period a risk-neutral agent faces an effort-substitution problem between work and search effort in the spirit of Holmström and Milgrom (1991). The agent's efforts cannot be observed by the principal. While work effort raises the principal's expected revenue, search effort improves the distribution over the agent's outside option that is randomly drawn at the beginning of the subsequent period. I assume that when the agent's outside option is sufficiently large, it is optimal to terminate the relationship. Search effort therefore raises the agent's expected continuation payoff by either getting him a better job or by improving his bargaining position vis-à-vis the principal. Since the location of a better job match is socially beneficial, OJS is not a pure rent-seeking activity. The principal offers spot contracts that specify a fixed wage and an output-contingent bonus that is enforceable. The agent is not protected by limited liability. Since search is socially valuable, the first-best allocation features a positive level of search effort. I focus on characterizing the *optimal incentive contract*, defined as the equilibrium sequence of spot contracts that maximizes the principal's payoff.

In a given period, the agent's returns from work effort are increasing in the current bonus, whereas the returns from OJS are decreasing in the rent that he expects to receive in the subsequent period. Full rent extraction by the principal distorts the agent's search incentives above their first-best level because, conditional on being retained, a higher outside option raises the agent's payoff but leaves the joint surplus unchanged. Conversely, first-best effort incentives require that the agent receive the full rent from trade. The principal can reduce the agent's

[3] Lemieux, MacLeod, and Parent (2009) document the highest levels of general pay and performance pay among highly educated workers and Cahuc, Postel-Vinay, and Robin (2006) find that skilled workers have higher degrees of bargaining power than unskilled workers. Further, Royalty (1998), Fallick and Fleischman (2004) and Nagypal (2005) document that OJS is more prevalent among highly educated workers.

incentives to search by implementing a voluntary wage floor such that the agent earns a rent whenever his outside option falls below this wage floor. However, because the returns from OJS depend on future rents, the principal faces a time-inconsistency problem regarding the implementation of this wage floor. A given wage floor is self-enforcing if the principal prefers to concede the implied rents over reverting to the full-rent-extraction equilibrium, thus creating a tradeoff between rent-extraction and efficiency. Under an interior solution of this tradeoff, the optimal incentive contract thus pays an efficiency wage. However, the purpose of the efficiency wage is different from its role in the classic model (Shapiro and Stiglitz (1984)). In particular, since the efficiency wage is paid to curb the agent's search incentives, it is not tied to a threat of dismissal and is paid unconditional on the agent's performance.

Since the principal never prefers to concede the full rent to the agent, the agent's search incentives are distorted upward relative to the first-best. This in turn makes it profitable to raise the performance-contingent bonus above its first-best level because, due to effort substitution, this reduces the agent's search activities. Hence, the optimal incentive contract for employed job seekers pays excessive bonuses in addition to the efficiency wage. This result also contrasts with the classic model where efficiency wages and bonus pay are are never used in conjunction (see MacLeod and Malcomson (1998)). Moreover, since the agent's total effort exceeds its first-best level, the optimal contract induces overworking.

Further, I show that if the contracting parties can renegotiate the incentive contract, then its structure will depend on the agent's initial-period outside option and will be renegotiated along the equilibrium path, which gives rise to non-trivial dynamics. In particular, the agent's earnings and his productivity are then increasing in his tenure, while the probability of a separation decreases as time passes. Both results are in line with well-established empirical regularities (see e.g. Topel and Ward (1992) and Farber (1999)), but they do not rely on the acquisition of skill-enhancing human capital or learning about match quality.

In section 7, I apply the framework to other contractual environments. First, I show that when output is unverifiable such that the payment of the bonus must also be self-enforcing, efficiency wages are even more valuable. Since a higher wage floor raises the joint surplus, it simultaneously increases the relationship value, thereby raising the maximum discretionary bonus that the principal can credibly promise to pay. Further, the paper's results are robust to the inclusion of a limited liability constraint. The bonus and the rent that the agent receives if he can conduct OJS are higher than in a limited-liability model without OJS. Finally, the paper's results prevail as long as the marginal costs of work (search) effort are increasing in the level of search (work) effort. When work and search effort are technologically independent, the optimal incentive contract pays an efficiency wage but the bonus is no longer excessive.

The next section relates the paper to the existing literature. Section 3 introduces the model setup. Section 4 characterizes the first-best solution and investigates the agent's incentive constraints. Section 5 derives the optimal incentive contract. Section 6 studies the properties of the optimal incentive contract under renegotiation. Section 7 applies the framework to other contractual environments. Section 8 concludes. All proofs are relegated to the Appendix.

2 Related Literature

While the impact of OJS on aggregate economic outcomes is well-studied[4], there have been few attempts to analyze how OJS affects strategic interactions in organizations. Moen and Rosen (2013) study a two-period principal-agent model with OJS and deferred compensation. In their model, OJS also mutes the agent's effort incentives, yet not because of multitasking, but because search effort reduces the likelihood that the agent will be present to collect his bonus.[5] Further, since they do not consider the effect of OJS on the agent's bargaining position, the agent always searches too little rather than too much. Also, while Moen and Rosen (2013) impose deferred compensation, it arises endogenously as part of the optimal contract in this paper. Board and Meyer-ter Vehn (2014) study how OJS shapes the market equilibrium when employers provide incentives via relational contracts. However, they focus on the effect of OJS on aggregate outcomes such as wage and productivity dispersion rather than on the details of the incentive contract. In addition, several authors study optimal long-term contracts in frictional labor markets when workers choose their search intensity endogenously (Postel-Vinay and Robin (2004), Holzner (2011) and Lentz (2014)). However, these papers restrict attention to the agent's search behavior. I complement this literature by investigating how the agent's search incentives interact with his work-effort incentives and how this affects the optimal incentive contract. Also, by analyzing the conditions under which efficiency wages are self-enforcing, I contribute a strategic foundation for the shape of long-term compensation contracts under OJS.

The paper also contributes to the growing literature that seeks to explain the recent surge in performance pay and pay levels in compensation contracts. Bénabou and Tirole (2013) develop a model in which competition for talented workers gives rise to a "bonus culture". They show that under asymmetric information about worker types, as competition intensifies, firms find it increasingly profitable to screen workers by distorting the performance incentives for high-skill workers above their first-best level. The present paper in contrast does not involve asymmetric information about worker characteristics. Rather, excessive bonuses are paid to reduce wasteful OJS by the agent. Hence, while in Bénabou and Tirole (2013), excessive bonuses are the cause of the inefficiency, in the present paper, excessive bonuses reduce the distortion created by the principal's reluctance to implement first-best search incentives.

The paper also adds to the literature on efficiency wages. In the classic shirking model of efficiency wages (Shapiro and Stiglitz (1984)), wage premiums motivate the worker because their payment is coupled with a threat of the worker being fired when caught shirking. Since these models study perfect monitoring environments, such inefficient separations occur only off equilibrium. Fuchs (2007) shows that motivating the agent via efficiency wages can also be optimal under private monitoring, since in this setting, incentive provision requires joint

[4]Burdett and Mortensen (1998) and Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) show that OJS can generate wage and productivity dispersion among ex-ante homogeneous jobs, while Pissarides (1994) studies search unemployment with OJS. Further, Lentz (2010) analyzes aggregate sorting patterns in the labor market under OJS and Lise (2013) studies how OJS and precautionary savings shape earnings inequality. For a survey of search-theoretic models of the labor market, including OJS, see Rogerson, Shimer, and Wright (2005).

[5]Moen and Rosen (2013) assume that work and search effort are technologically independent.

punishment. A central takeaway from these two strands of the literature is that, in equilibrium, efficiency wages and performance pay are never used jointly to motivate the agent. MacLeod and Malcomson (1998) show under perfect monitoring that an efficient market will select either efficiency wages or bonus pay as the unique incentive device, depending on whether there is excess supply of, or excess demand for, labor. Under subjective performance evaluations, Maestri (2012) shows that whether efficiency wages or bonus pay should be used depends on players' patience and the correlation of their performance signals. Finally, under imperfect monitoring, bonus pay always dominates efficiency wages as their use would lead to inefficient separations on the equilibrium path. This paper provides an alternative perspective on the use of efficiency wages in incentive contracts that overturns some of these results. Efficiency wages lower workers' incentives to perform wasteful OJS and are therefore not tied to a firing threat and are paid unconditional on the agent's performance. Thus, the paper's results do not only rationalize the use of efficiency wages in imperfect monitoring environments, they also offer a justification for why firms may want to use a combination of efficiency wages and bonus pay.[6]

The analysis in the paper applies insights from the literature on multitask-agency settings. In their seminal contribution, Holmström and Milgrom (1991) show that the agent's incentives to perform a given task depend not only on the returns from that task, but also on its opportunity costs which are determined by the returns from other tasks. In this paper, the agent's returns from work effort are given by the current bonus, whereas the returns from job search depend on his expectations about future contracts. Therefore, multitasking between these two tasks renders the problem of designing the optimal incentive contract dynamic. Another key takeaway from Holmström and Milgrom (1991) is that incentives are weaker in multitasking settings, when the performance in some tasks is more difficult to measure than in others. By contrast, the multitask setting in this paper gives rise to excessive performance pay, because the principal prefers to reduce the agent's search incentives by raising its opportunity costs.

Furthermore, the paper is related to the literature on hold-up problems (Hart and Moore (1988)), where it is also one party's ex-post incentive to exploit a weak bargaining position of the other party that distorts ex-ante incentives. Baker, Gibbons, and Murphy (2002) and Gibbons (2005) analyze relational contracts in a setting where an agent performs a productive activity that raises the joint surplus and a rent-seeking activity that improves his bargaining position in the ex-post negotiation over the division of the created surplus. By contrast, in the present paper, OJS improves the agent's bargaining position in the negotiations over the continuation surplus. Therefore, by influencing the agent's continuation behavior, the principal's contract offer does not only determine the division, but also the size of the continuation surplus. Also, while in Baker, Gibbons, and Murphy (2002) and Gibbons (2005), the inefficiency is due to parties' inability to contract ex-ante on ex-post outcomes, I show that under OJS, the distortion persists even though parties can write fully enforceable spot contracts. Further, Inderst (2014)

[6]Evidently, under limited liability, optimal incentive contracts also combine performance pay and rent payments (see e.g. Laffont and Martimort (2009)). However, in these models, the principal is forced to pay a rent and does not deliberately decide to do so in order to raise her profits. Further, in informed-principal models, the principal may in fact prefer to leave the agent a rent in addition to a bonus in order to signal his type (see e.g. Beaudry (1994)).

studies how ex-post renegotiation of a manager's employment contract distorts his ex-ante incentives to choose the right continuation strategy for a firm. He shows that this distortion can be mitigated by contracts that offer job protection. In a closely related paper, Giannetti (2011) shows that, in order to induce a manager to choose a long-term strategy rather than an inferior short-term strategy, the principal must commit to paying him a sufficiently high share of the future surplus. I complement these papers by showing that in a general moral-hazard model in which the agent performs OJS, similar economic forces lead to several non-standard properties of the optimal incentive contract.

Further, a huge literature models employment as an ongoing principal-agent relationship (e.g. Radner (1985) and Malcomson and Spinnewyn (1988)). Models of relational contracts investigate how repeated interactions can help parties sustain self-enforcing incentive contracts (key references include Bull (1987), MacLeod and Malcomson (1989) and Levin (2003), Malcomson (2012) provides a comprehensive survey.). First, I add to this literature the insight that when the agent performs OJS, continued trade enables the principal to offer more efficient incentive contracts by complementing formal bonus contracts with an informal promise to pay an efficiency wage. Second, I contribute to the branch of the literature that derives non-trivial dynamics in repeated principal-agent models (e.g. Chassang (2010), Halac (2012) and Li and Matouschek (2013)). I show that under OJS, optimal incentive contracts may give rise to wages and productivity that increase with the agent's tenure, as well as to a turnover rate that decreases with tenure.

Finally, this is not the first paper that endogenizes the agent's outside option in a multiperiod principal-agent model. Englmaier, Muehlheusser, and Roider (2014) study optimal incentives for "knowledge workers" whose efforts for one principal simultaneously raises the value of their services for other employers. Further, Netzer and Scheuer (2010) analyze a setting where ex-ante investments by agents determine their type in an ex-post matching market. However, in those papers, it is the same action that raises both, the agent's outside option as well as the principal's revenue. Therefore, these papers naturally generate very different implications regarding optimal incentive provision than this paper.

3 Model

Setup A principal (she) and an agent (he) can trade over an infinite time horizon with dates $t = 1, 2, ...$. Both players are risk-neutral and discount payoffs with the factor $\delta \in (0, 1)$. At the beginning of every period t, nature draws the agent's outside option r_t from the interval $[\underline{r}, \overline{r}]$. The principal's outside option $\overline{\pi}$ is the same in every period. After both players have observed r_t, the principal makes a contract offer to the agent whose participation decision is denoted $d_t \in \{0, 1\}$. If the agent accepts the offer ($d_t = 1$), he subsequently exerts work effort $e_t \in [0, \overline{e}]$ and search effort $s_t \in [0, \overline{s}]$ at private cost $c(e_t + s_t)$, with $c(0) = 0$, $c'(.) > 0$, $c''(.) > 0$ and $c'(\overline{e}) = c'(\overline{s}) = +\infty$. I call (e_t, s_t) the agent's *effort profile*. The principal can neither observe e_t nor s_t.

Work effort generates a stochastic output $y_t \in \{\underline{y}, \overline{y}\}$ for the principal. Given e_t, the probability that $y_t = \overline{y}$ is $f(e_t) \in (0,1)$, with $f'(.) > 0$ and $f''(.) \leq 0$. Output is publicly observable and verifiable to third parties. The expected joint surplus created in period t is denoted by $T(e_t, s_t) = \mathbb{E}_y(y_t|e_t) - c(e_t + s_t)$.

Search effort in period t determines the probability distribution of r_{t+1}. More precisely, r_t is drawn according to the distribution function $G(r_t|s_{t-1})$, with $G_s(r|s) \leq 0$ and $G_{ss}(r|s) \geq 0$, where $G_s(.|.)$ and $G_{ss}(.|.)$ denote the partial derivatives of $G(.|.)$ with respect to s. So, in any period, the distribution of the agent's outside option given some search effort level in the previous period dominates (is dominated by) any distribution associated with a lower (higher) previous-period search effort level in the sense of (weak) first-order stochastic dominance.[7] I assume that G has full support for any $s > 0$.[8]

If the agent rejects the principal's offer at date t ($d_t = 0$), the game ends ($d_\tau = 0$ for all $\tau > t$) and both players receive their period-t outside options in perpetuity.[9,10] I impose the following condition on the relative productivity of the relationship:

Assumption A1.

$$\overline{r} + \overline{\pi} > \max_{(e,s)} T(e, s).$$

A1 states that there exist other jobs in which the agent is more productive than in the principal's job. However, due to frictions in the labor market, the agent must exert costly search effort to locate them. Hence, under A1, search effort has a social value.

Contracting The principal's contract offer at date t specifies a fixed wage w_t and an output-based bonus b_t. The fixed wage is paid conditional on $d_t = 1$ and, without loss of generality, the principal pays a high bonus $\overline{b}_t$ if $y_t = \overline{y}$ and a low bonus $\underline{b}_t$ if $y_t = \underline{y}$. The agent is not protected by limited liability, implying that total compensation $W_t \equiv w_t + b_t$ can be negative. Hence, there is no loss of generality in setting $\underline{b}_t = 0$ and $\overline{b}_t = b_t \geq 0$. Both components of the contract are enforceable. Thus, there is no commitment problem regarding the payment of the bonus. However, players cannot contract ex ante on the agent's outside option in future periods. In Section 7, I study both more complete (long-term) contracting as

[7]The assumption that the second derivative of $G(.|.)$ with respect to s is non-negative is not a requirement of first-order stochastic dominance. I impose it to ensure that the marginal returns from search effort are decreasing, i.e. the distribution of r_t improves at a decreasing rate, as s_{t-1} increases.

[8]Assuming that the agent draws a new outside option in every period means that he cannot recall any offers that he rejected in the past.

[9]Note that r_t does not merely reflect the wage that the agent can get in the alternative job. See the subparagraph "Payoffs" for a discussion.

[10]Instead of assuming that the game ends when $d_t = 0$, one could allow the principal and the agent to meet again in the period after the agent has rejected the principal's offer. In that case, it would be natural to assume that the agent carries r_t over to date $t+1$ i.e. $r_{t+1} = r_t$ whenever $d_t = 0$. Under these assumptions, the paper's results would remain unchanged since in equilibrium, the agent would never accept an offer by the principal at date τ if he has rejected her offer at some prior date $t < \tau$. Further, it would be inconsistent to assume that the agent cannot recall past outside options from alternative employers, but can always come back to the principal. Finally, in the studied environment, temporary separations off the equilibrium path do not enhance the set of equilibrium payoffs.

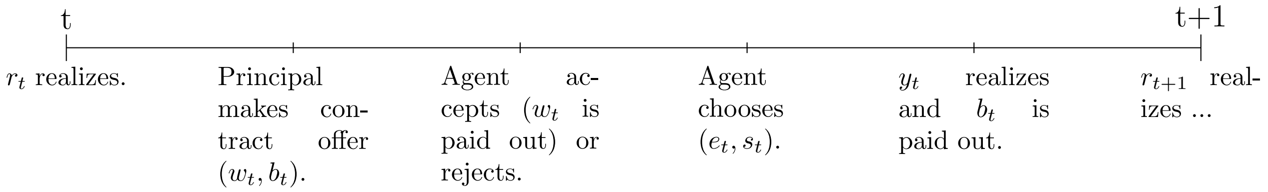

Figure 1: Sequence of events.

well as more incomplete (b_t not enforceable) contracting environments. Figure 1 illustrates the timing of events in a representative period.

Payoffs The expected payoffs for the principal and the agent at date t are

$$\pi_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty}[d_\tau(y_\tau - W_\tau) + (1-d_\tau)\overline{\pi}]\right],$$

$$u_t = (1-\delta)\mathbb{E}\left[\sum_{\tau=t}^{\infty}[d_\tau\left(W_\tau - c(e_\tau + s_\tau)\right) + (1-d_\tau)r_\tau]\right],$$

respectively. Let $Q_t \equiv \pi_t + u_t$ denote the joint surplus. Thus, while $T(e_t, s_t)$ denotes the period-surplus that is created *at* date t, Q_t represents the per-period surplus of the supergame *from* period t *onward*. Hence, Q_t contains not only the surplus produced by principal and agent in periods in which they trade with each other, but also the sum of the payoffs generated by each player individually at dates $\tau > t$ with $d_\tau = 0$. Further, since all payoffs are expressed as per-period averages, $r \in [\underline{r}, \overline{r}]$ represents a reservation payoff that reflects the value of the job associated with r. In particular, r comprises both the wage that the agent can earn in the job, as well as the option value implied by the agent's expected search activities in the job. The lower bound $\underline{r}$ can thus be interpreted as the value of being unemployed. The Appendix sketches a model of a labor market with heterogeneous firms in which $[\underline{r}, \overline{r}]$ is endogenously determined by firms' equilibrium behavior.

Equilibrium Concept I study perfect public equilibria (PPE) of the repeated game. In a PPE, players condition their strategies only on public histories and, after any public history, their strategies must constitute a Nash equilibrium. (Fudenberg, Levine, and Maskin (1994)).[11] Specifically, I look for PPE that maximize the principal's payoff and I define an *optimal incentive contract* as the sequence of contract offers $\{(w_t, b_t)\}_{t=1}^{\infty}$ associated with such a PPE.

[11]Restricting attention to public strategies is without loss of generality. The agent does have private information about the effort profile. However, since this kind of private information is one-sided, the outcome of any equilibrium in which players use private strategies is also the outcome of an equilibrium where players use public strategies, see Mailath and Samuelson (2006).

4 Preliminaries

4.1 Equilibrium Conditions

Let $\pi_t(r_t, d_t)$ and $u_t(r_t, d_t)$ denote the principal's and the agent's continuation payoffs at date t under some public strategy profile, as functions of r_t and d_t. As PPE requires players' participation decisions to be individually rational, the following participation constraint must be satisfied:

$$\text{For any date } t\text{, } d_t = 1 \text{ if and only if } \pi_t(r_t, 1) \geq \overline{\pi} \text{ and } u_t(r_t, 1) \geq r_t. \tag{PC}$$

Since $G(r|s)$ has full support for any $s > 0$, (PC) and A1 imply that the agent will eventually draw an outside option that triggers separation. Specifically, call ρ_t such that $d_t = 1$ if and only if $r_t \leq \rho_t$, the *separation threshold.*

Lemma 1. *For any public history occurring on the equilibrium path of some PPE, $\rho_t < \overline{r}$.*

Lemma 1 implies that when the agent exerts search effort, he will change jobs with strictly positive probability on the equilibrium path. These separations however are voluntary as the agent leaves because he can get a payoff in another job that is higher than the maximum payoff that the principal is willing to pay him.

Further, in any PPE, whenever $d_t = 1$, (e_t, s_t) maximizes the agent's expected payoff:

$$(e_t, s_t) \in \arg\max_{e,s} \; (1 - \delta) \left[\mathbb{E}_y \left(W_t|e\right) - c(e + s)\right] + \delta \mathbb{E}_r \left(u_{t+1}|s_t\right). \tag{IC}$$

(IC) represents the hidden action problem. The agent chooses work and search effort optimally, given the current bonus and his expectations about his continuation payoff.

4.2 Benchmarks

No Search Note that the model involves no frictions such as risk aversion, limited liability or incomplete contracts that create distortions in standard agency models without search effort. Hence, in the absence of search effort ($s_t = 0$ for all t), the principal would maximize her profits by setting $b_t = \Delta y$ for every t, thus implementing $\max_{(e,s)} T(e, s)$ and adjusting the fixed wage to extract the full employment rent.

First-best Let Q_t^{FB} denote the first-best (or maximum) joint surplus at date t and let $\left\{(e_t^{FB}, s_t^{FB})\right\}_{t|d_t=1}$ denote the corresponding sequence of first-best effort profiles.

Lemma 2. *The first-best effort profile and the first-best surplus are stationary: $(e_t^{FB}, s_t^{FB}) = (e^{FB}, s^{FB})$ and $Q_t^{FB} = Q^{FB}$ for all t with $d_t = 1$. Further, the first-best separation threshold is also stationary and given by: $\rho^{FB} = Q^{FB} - \overline{\pi}$ for all r_t.*

Stationarity follows from the fact that, conditional on $d_t = 1$, the technological environment is identical in every period. Therefore, the first-best joint surplus (in any period with $d_t = 1$)

can be expressed as a weighted average of the joint surplus generated per period in which the relationship is continued and the expected sum of the payoffs that principal and agent receive individually after having separated:

$$Q^{FB} = \phi T(e^{FB}, s^{FB}) + (1-\phi)\left[\mathbb{E}_r\left(r|r > \rho^{FB}, s^{FB}\right) + \overline{\pi}\right], \text{ where } \phi = \frac{1-\delta}{1-\delta G(\rho^{FB}|s^{FB})}. \quad (1)$$

Under an interior solution ($e^{FB}, s^{FB} > 0$), this expression can be used to obtain the first-order conditions characterizing the first-best solution:[12]

$$f'(e)\Delta y = c'(e+s), \quad (FB_e)$$

$$\frac{\delta}{1-\delta}\left\{G_s(\rho^{FB}|s)\phi\left[T(e,s) - \overline{\pi} - \mathbb{E}_r(r|r > \rho^{FB}, s)\right] + \left[1 - G(\rho^{FB}|s)\right]\frac{\partial \mathbb{E}_r\left(r|r > \rho^{FB}, s\right)}{\partial s}\right\}$$
$$= c'(e+s). \quad (FB_s)$$

This program has an interior solution if the effect of search effort on $G(r_{t+1}|s_t)$ at $s_t = 0$ is sufficiently strong, i.e. if the marginal social value of search at $s_t = 0$ is sufficiently large.[13] While search effort reduces the current-period surplus, it increases the joint continuation surplus first, by raising the likelihood that the agent locates a better job in the next period, and second, by raising the expected quality of the agent's new job conditional on separation. Since I am interested in the incentives of employed job seekers, I focus on the case where the first-best level of search effort is positive, as this is a sufficient condition for the agent's equilibrium level of search effort to be positive.

In the absence of a hidden action problem, i.e. when (e,s) is contractible, the principal maximizes her payoff by offering a forcing contract that implements (e^{FB}, s^{FB}) and that extracts the full employment rent by paying the agent a fixed wage equal to:

$$w^{FB} = \frac{r}{1-\delta} + c(e^{FB} + s^{FB}) - f(e^{FB})b^{FB} - \frac{\delta}{1-\delta}\mathbb{E}(r|s^{FB}).$$

Intuitively, the principal benefits from a positive level of search effort because it improves the agent's future job prospects, thereby increasing his payoff from the employment relationship. In turn, this reduces the fixed payment necessary to induce the agent to participate and thus allows the principal to raise her payoff.[14]

[12] The second-order sufficient conditions for the first-order conditions to represent the problem's unique global maximum can fail only in very special cases. The Appendix provides sufficient conditions that rule out such cases. In general, the first-order conditions represent the problem's unique global maximum if $c(.)$ or $G(.|.)$ or both are sufficiently convex.

[13] The precise necessary and sufficient condition for an interior solution is:

$$\frac{\delta}{1-\delta}\left\{G_s(\rho^{FB}|s)\phi\left[T(e,s) - \overline{\pi} - \mathbb{E}_r(r|r > \rho^{FB}, s)\right] + \left[1 - G(\rho^{FB}|s)\right]\frac{\partial \mathbb{E}_r\left(r|r > \rho^{FB}, s\right)}{\partial s}\right\}_{|(e,s)=(e^{FB},0)}$$
$$> f'(e^{FB})\Delta y = c'(e^{FB}).$$

[14] Again, this does not imply that the agent is unable to conduct OJS in other jobs. For instance, when the

4.3 Incentive Provision

Under hidden action, the agent chooses (e_t, s_t) according to (IC). Since work and search effort are (perfect) substitutes in the agent's cost function, he faces an effort-substitution problem. Holmström and Milgrom (1991) were the first to study optimal incentive provision when agents can engage in multiple tasks. In such a setting, the agent's incentives to execute a given task depend not only on the direct payoff consequences from performing that task, but also on its opportunity costs, given by the returns from performing other tasks that compete for the agent's attention. To identify the returns from work and search effort, consider the first-order conditions of his problem:[15]

$$f'(e_t)b_t = c'(e_t + s_t), \tag{IC_e}$$

$$\frac{\delta}{1-\delta}\frac{\partial E_r\left(u_{t+1}|s_t\right)}{\partial s_t} = c'(e_t + s_t). \tag{IC_s}$$

Since the marginal costs are the same, the agent chooses (e_t, s_t) to equate the marginal returns from work and search effort. These are given by the left-hand sides of (IC_e) and (IC_s).[16] The agent's behavior at date t depends both on the current contract as well as on his expectations about future contract offers by the principal. The returns from work effort are determined by the bonus rate specified in the current contract. As in standard moral-hazard models, a larger bonus raises the returns from work effort. But because the agent faces an effort-substitution problem, a larger b_t simultaneously increases the opportunity costs of search effort. Denoting the agent's optimal decision by (e_t^A, s_t^A), (IC_e) and (IC_s) imply:

$$\frac{\partial e_t^A}{\partial b_t} > 0, \frac{\partial s_t^A}{\partial b_t} \leq 0 \text{ and } \frac{\partial e_t^A + s_t^A}{\partial b_t} > 0. \tag{2}$$

The marginal returns from search effort in turn depend on how the change in the distribution of r_{t+1} induced by s_t affects the agent's expected continuation payoff from $t+1$ onward. To understand the economic forces that drive this effect, rewrite the marginal change in $E_r\left(u_{t+1}|s_t\right)$ as:

$$\frac{\partial E_r\left(u_{t+1}|s_t\right)}{\partial s_t} = -G_s(\rho_{t+1}|s_t)\left[E_r\left(r|s_t; r > \rho_{t+1}\right) - E_r\left(u_{t+1}|s_t; r \leq \rho_{t+1}\right)\right]$$
$$+\left[1 - G(\rho_{t+1}|s_t)\right]\frac{\partial E_r\left(r|s_t; r > \rho_{t+1}\right)}{\partial s_t} + G(\rho_{t+1}|s_t)\frac{\partial E_r\left(u_{t+1}|s_t; r \leq \rho_{t+1}\right)}{\partial s_t}.$$

First, search effort raises the likelihood that the agent locates a better job (first term). Finding a better job is more desirable, the smaller the agent's payoff under retention. Second, conditional on changing jobs, search effort increases the expected quality of the agent's new job (second

agent is unemployed ($r_t = \underline{r}$), he will typically search more than in any job $r_t \neq \underline{r}$. Thus, allowing some OJS relaxes the agent's participation constraint by reducing the "relative search-advantage" of other jobs in which the agent expects to search more than in his current job.

[15] Recall that I consider the case where the agent's problem has an interior solution. Corner solutions involving $s_t = 0$ are discussed in Section 5.3.

[16] Again, this is assuming that there exists an interior solution. The assumptions on $f(.)$, $c(.)$ and $G(.|.)$ ensure that (IC_e) and (IC_s) represent the agent's problem's unique global maximum.

term). These two components constitute the *job-switching motive* of OJS. Via the first term, the job-switching motive depends on the payoff the agent receives under retention. Third, search effort raises the expected surplus-share the agent can extract conditional on being retained (third term). I call this component the *rent-seeking motive* of OJS. The rent-seeking motive depends on how the principal tailors her contract offer to the agent's outside option. If she never leaves a rent to the agent ($u_{t+1}(r_{t+1}, 1) = r_{t+1}$), the rent-seeking motive is very strong.

First-Best Effort Incentives The next result characterizes the sequence of spot contracts that implements the first-best effort profile.

Lemma 3. $(e_t^A, s_t^A) = (e^{FB}, s^{FB})$ *for all* t *with* $d_t = 1$ *if and only if* $d_t = 1$, $b_t = \Delta y$ *and* $u_t(r_t, 1) = \rho^{FB}$ *for all* $r_t \leq \rho^{FB}$ *and* $d_t = 0$ *and* $u_t(r_t, 0) = r_t$ *for all* $r_t > \rho^{FB}$.

Since ρ^{FB} represents the principal's maximum willingness-to-pay for the agent's services, Lemma 3 implies that the first-best effort profile can only be implemented if the agent receives the full employment rent. Intuitively, the agent's private marginal returns from work and search effort must coincide with the corresponding marginal change in joint surplus. Recall that search effort affects the joint surplus by raising the likelihood of a profitable separation and the expected quality of the agent's job under separation. Yet, conditional on retention, the joint surplus is independent of r_t and hence of s_{t-1}. Therefore, first-best incentives require that the agent's payoff under retention is also independent of r_t. Since by (PC), $u_t(r, 1) \geq r$ for any $r \leq \rho^{FB}$, this can only be achieved if $u_t(r, 1) = \rho^{FB}$ for every $r \leq \rho^{FB}$.

A Time-Inconsistency Problem As players can only write spot contracts, the principal faces a time-inconsistency problem regarding the provision of effort incentives. In particular, it is b_t and the prospect of future rents that determine the agent's behavior in period t. Thus, at any date t, the principal has an incentive to withhold the rent she promised at some earlier date. In a PPE however, the agent anticipates this behavior and hence chooses e_t and s_t, given $u_{t+1}(r, d_{t+1}) = r$ for any r. To make rent payments self-enforcing, the agent must react to a failure by the principal to pay a previously promised rent. In fact, a rational agent should adjust his expectations and hence his behavior in response to such a deviation by the principal. To see this, suppose that the principal promises to pay a rent for certain realizations of r_t. If the principal sticks to her promise, there is no reason for the agent to believe that she will defect in future periods. However, whenever the principal fails to pay a promised rent, the agent should become pessimistic about the principal's willingness to honor her promises in future periods. If the change in the agent's behavior induced by such an adjustment of expectations leads to a sufficiently strong reduction in the principal's payoff, she will prefer to pay the rent. Hence, in order to derive the principal's propensity to pay a rent, it is necessary to know equilibrium play under pessimistic expectations. I therefore begin the analysis of optimal incentive provision by characterizing the optimal incentive contract under the restriction that the agent earns no rent from employment.

5 Equilibrium Analysis

5.1 Full Rent Extraction

In this subsection, I restrict attention to PPE in which $u_t = r_t$ for all t with $d_t = 1$. To see that there always exists such a PPE, suppose that the agent expects to receive $u_{t+1} = r_{t+1}$ for any r_{t+1}, independent of the principal's offer in t. In that case, the principal's best response to the implied effort profile by the agent is evidently to leave him no rent. Thus, the PPE with full rent extraction by the principal represents the unique PPE when players are myopic in the sense that their behavior is independent of the public history of play.[17] Under full rent extraction, (IC_s) becomes:

$$\frac{\delta}{1-\delta}\frac{\partial E_r\left(r_{t+1}|s_t\right)}{\partial s_t} = c'(e_t + s_t). \qquad (IC_s^{NR})$$

When the principal keeps the agent to his reservation payoff, search effort raises the agent's continuation payoff both conditional on being retained and conditional on leaving for another job. However only the latter effect represents an actual increase in joint surplus because, conditional on separation, a better outside option represents a better job match. By contrast, the increase in the agent's payoff under retention merely reflects a redistribution of surplus. Hence, under full rent extraction, the agent's private returns from search effort are strictly larger than the joint returns.[18] Further, since the agent's continuation payoff is determined by his outside option r_t, the principal can influence the agent's behavior solely via the bonus. Let $\left\{(w_t^{NR}, b_t^{NR})\right\}_{t=1}^{\infty}$ denote the optimal incentive contract under full rent extraction. The principal thus chooses $\left\{(w_t^{NR}, b_t^{NR})\right\}_{t=1}^{\infty}$ to maximize the joint surplus subject to $u_t = r_t$, (IC_e) and (IC_s^{NR}).

Lemma 4. *Under full rent extraction by the principal, the optimal incentive contract ...*

(i) ... is stationary: $(w_t^{NR}, b_t^{NR}) = (w^{NR}(r_t), b^{NR})$ *for all* t *with* $d_t = 1$.

(ii) ... does not maximize the joint surplus: $Q^{NR} < Q^{FB}$.

(iii) ... features excessively steep performance incentives: $b^{NR} > \Delta y$.

The following paragraph develops the intuition behind Lemma 4. Let Q_e and Q_s denote the marginal effects of e and s on the joint surplus. Since the principal can always adjust the fixed wage to satisfy PC and because the technological environment is stationary, the surplus-maximizing effort profile must also be stationary.[19] Further, for any level of e, the agent exerts more search effort than is optimal from the principal's perspective. Thus, $Q_s < 0$ for any (e, s), implying that for any implemented effort profile, the principal prefers to reduce s. However,

[17] Notice that the supergame does not have a well-defined stage game, because the consequences of search effort in a given period occur in the next period.

[18] The Appendix provides a formal derivation of this result.

[19] The contract characterized in Lemma 4 is stationary in the sense that the bonus rate is constant and the function $w(r_t)$ that maps the agent's outside option into his fixed wage is the same for any date t.

under full rent extraction, s can only be reduced by raising the bonus which simultaneously increases e. As long as work effort raises the joint surplus (which is the case for $b < \Delta y$), the principal therefore strictly prefers to raise the bonus. At $b = \Delta y$, the principal and the agent have aligned interests regarding the choice of e, while increasing b above Δy generates excessive work effort incentives ($Q_e < 0$). However, at $b = \Delta y$, the reduction in total surplus caused by a marginal increase in b is of second order and therefore outweighed by the first-order increase in surplus due to the simultaneous reduction in s. Therefore, the optimal bonus payment under full rent extraction is higher than the first-best bonus. The condition that characterizes the optimal bonus payment when $u_t = r_t$ for all t is

$$Q_e \frac{\partial e^A}{\partial b} + Q_s \frac{\partial s^A}{\partial b} = 0 \Leftrightarrow -Q_e \frac{\partial e^A}{\partial b} = Q_s \frac{\partial s^A}{\partial b} > 0, \text{ with } Q_e < 0 \text{ and } Q_s < 0.^{20} \tag{3}$$

Thus, when the principal extracts the full employment rent, joint surplus (and the principal's payoff) is maximized when the marginal benefits of a further reduction in s generated by a marginal increase in b ($Q_s \frac{\partial s^A}{\partial b}$) are just offset by the additional costs implied by the corresponding increase in e ($-Q_e \frac{\partial e^A}{\partial b}$).

5.2 Efficiency Wages

As shown in the previous subsection, full rent extraction by the principal distorts the agent's search effort incentives above their first-best level. More generally, as long as the agent does not receive the full rent from employment, his rent-seeking motive is positive, because higher outside options increase his payoff conditional on retention. In addition, the agent also has an excessive job-switching motive, because the value he derives from a job switch is higher than the corresponding first-best value. To reduce the rent-seeking motive, the principal must make the agent's payoff under retention respond less strongly to variations in his outside option. Conversely, the job-switching motive can be reduced by raising the agent's expected payoff conditional on being retained. A strategy that reduces both motives at once is one under which the principal never lets the agent's payoff fall below some voluntary wage floor $r_0 > \underline{r}$ such that:

$$u_t(r,1) = \begin{cases} r_0, & \text{if } r \leq r_0, \\ r, & \text{if } r > r_0. \end{cases} \tag{4}$$

Under such a strategy, the principal must give up a rent whenever $r_t < r_0$.[21] Notice that the principal's wage policies under the first-best and under full rent extraction are both special cases of (4). While first-best effort incentives require $r_0 = \rho^{FB}$, under full rent extraction $r_0 = \underline{r}$. It is straightforward to verify that the wage floor raises the agent's expected payoff conditional on being retained, and reduces the marginal effect of s_t on it.[22] Thus, (IC_e) and

[20] The assumptions on the production technology ensure that the first-order approach is valid.

[21] Given $\{b_t\}_{t=1}^{\infty}$, the principal can implement this strategy by offering the fixed wage $w_t = \frac{r_0}{1-\delta} - f(e_t)b_t + c(e_t + s_t) - \frac{\delta}{1-\delta}\mathbb{E}(u_{t+1})$ for any $r_t \leq r_0$.

[22] This class of strategies need not be optimal. In general, the optimal wage policy depends on the precise specification of the function $G(.|.)$. However, the derivation of the optimal use of the rent as a function of the

(IC_s) imply

$$\frac{\partial e_t^A}{\partial r_0} \geq 0, \frac{\partial s_t^A}{\partial r_0} < 0 \text{ and } \frac{\partial e_t^A + s_t^A}{\partial r_0} < 0. \quad (5)$$

Hence, a higher wage floor has the same qualitative effect on the agent's behavior as an increase in the bonus rate. However, the economic forces through which the two instruments work are different. While a higher bonus rate raises the returns from work effort, a higher wage floor reduces the returns from search effort.[23] Intuitively, under a wage floor $r_0 > \underline{r}$, the agent is not as desperate to avoid low outside options as under full rent extraction. However, as the next result shows, b and r_0 have different qualitative effects on the agent's effort choice.

Corollary 1. *For any (e_t, s_t), we have*

$$\frac{\partial e_t^A/\partial b_t}{|\partial s_t^A/\partial b_t|} > 1 > \frac{\partial e_t^A/\partial r_0}{|\partial s_t^A/\partial r_0|}.$$

Corollary 1 states that a unit-reduction in search effort obtained by raising the wage floor is associated with a smaller increase in work effort than an identical reduction in s_t generated by raising the bonus. Now, recall that under the full-rent-extraction optimum ($b_t = b^{NR}$ and $r_0 = \underline{r}$), the marginal benefits of reducing s by raising b further are exactly offset by the marginal costs due to the corresponding rise in e. But by Corollary 1, lowering s through r_0 entails a smaller relative increase in e. Hence, raising r_0 above $\underline{r}$ at b^{NR} increases the joint surplus. Put differently, because $b > b^{FB}$ implies $e^A > e^{FB}$, a given reduction in s can be achieved more cheaply by raising the wage floor rather than the bonus. Let $b(r_0)$ denote the bonus payment that maximizes the principal's payoff (and at the same time the joint surplus) conditional on r_0 and let $Q(r_0)$ denote the resulting joint surplus. We have the following result:

Proposition 1. *For any $r_0 \in \left[\underline{r}, \rho^{FB}\right)$, the joint surplus is strictly increasing in r_0, i.e. $Q'(r_0) > 0$, and the principal-payoff-maximizing bonus is strictly larger than the first-best bonus, i.e. $b(r_0) > \Delta y$.*

Intuitively, as long as $r_0 < \rho^{FB}$, the agent's returns from search effort are higher than the joint returns. For similar reasons as under full rent extraction, for any $r_0 < \rho^{FB}$, it is then profitable to have $b(r_0) > \Delta y$. But then by Corollary 1, the joint surplus can be increased by raising the wage floor r_0 at the margin.

Self-Enforcing Rent Payments Proposition 1 shows that OJS by an agent generates a tradeoff between rent-extraction and efficiency. While a higher wage floor raises the joint surplus, the principal has to give up a rent whenever $r_t < r_0$. However, as discussed in Section 4.3, rent payments must be self-enforcing to affect the agent's effort incentives.

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details of $G(.|.)$ goes beyond the scope of the paper. I focus on strategies of the form given by (4), because these represent the unique class of strategies that reduces both, the rent-seeking and the job-switching motive for any specification of $G(.|.)$. See the Appendix for a formal argument.

[23] Note that this result does not depend on work and search effort being perfect substitutes in the agent's cost function. All that is needed is that the marginal costs of work (search) effort are increasing in the level of search (work) effort. See Section 7 for a formal argument.

I study strategies under which the failure of the principal to pay a promised rent induces the agent to become pessimistic about the principal's willingness to give up rents in future periods. Under such a strategy, a failure to pay a promised rent triggers play of the PPE characterized in Lemma 4. It is natural to assume that players revert to the full-rent-extraction equilibrium, as it represents the unique PPE that exists for any level of δ and any specification of $G(.|.)$. Under such a strategy profile, a given voluntary wage floor is self-enforcing if and only if the principal has sufficient incentives to pay r_0 whenever called upon to do so, i.e. if and only if

$$Q(r_0) - r_0 \geq Q^{NR} - r, \ \forall r \in [\underline{r}, r_0) \, .$$

As this condition is most stringent for $r = \underline{r}$, it holds if and only if the following *enforcement constraint* is satisfied:

$$Q(r_0) - r_0 \geq Q^{NR} - \underline{r} \Leftrightarrow Q(r_0) - Q^{NR} \geq r_0 - \underline{r}. \qquad (EC)$$

Thus, a given wage floor $r_0 > \underline{r}$ is self-enforcing if and only if the increase in the surplus it generates exceeds the (maximum) wage costs it implies. Notice that this is precisely the definition of an efficiency wage (see e.g. Carmichael (1990)). Further, (EC) implies that the principal's payoff in any PPE with $r_0 > \underline{r}$ is higher than in the PPE in which she extracts the full rent. It follows that the optimal incentive contract pays an efficiency wage whenever these exist, i.e. whenever voluntary wage floors are self-enforcing. The next result provides a necessary and sufficient condition for the existence of PPE in which $r_0 > \underline{r}$:

Lemma 5. (EC) *holds for a nonempty subset of* $r_0 \in \left(\underline{r}, \rho^{FB}\right]$ *if and only if:*

$$Q'(r_0 = \underline{r}) \geq 1. \qquad (EW)$$

If (EW) holds, every $r_0 \in [\underline{r}, r_0^{max}]$ can be implemented in some PPE, where r_0^{max} is defined as the wage floor at which (EC) binds. Moreover, this interval is larger the greater the agency costs in the PPE in which the principal extracts all gains from trade. This relationship is illustrated in Figure 3.

Further, notice that, because $Q^{NR} - \underline{r} > \overline{\pi}$, the right-hand side of (EC) is always greater than $\overline{\pi} = Q^{FB} - \rho^{FB}$.[24] The next result readily follows.

Proposition 2. *There exists no PPE that implements the first-best, i.e.* $r_0^{max} < \rho^{FB}$.

Thus, the principal's time-inconsistency problem makes it impossible to implement first-best effort incentives. Intuitively, if there are gains from trade when the principal extracts all of them, she can never be induced to give up all gains from trade in another PPE.

[24]Intuitively, under full rent extraction, changes in the agent's outside option r_t translate one-to-one into his continuation payoff u_t. Therefore, the agent's returns from search effort under full rent extraction are just as high as under unemployment in which case his payoff is $\underline{r}$. However, because the agent also produces some output in the job, we must have $Q^{NR} > \overline{\pi} + \underline{r}$.

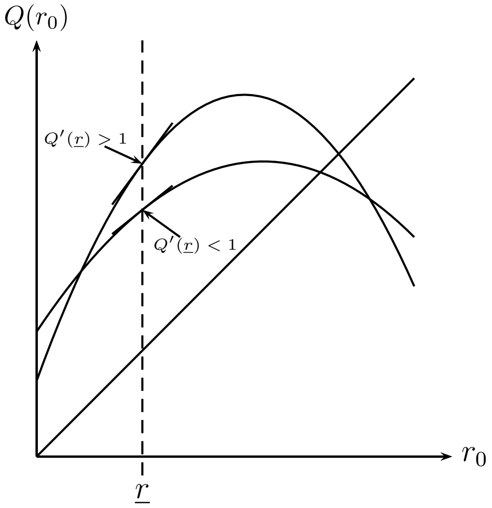
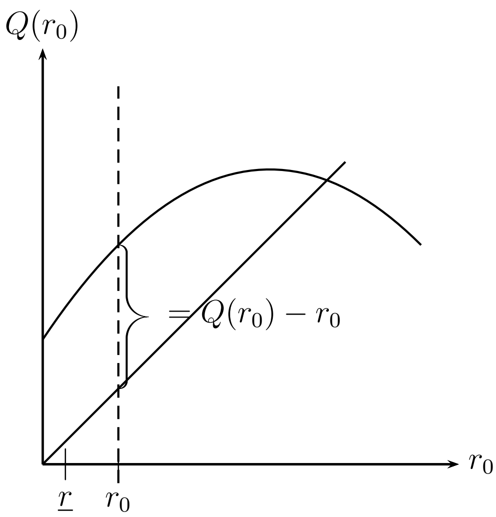
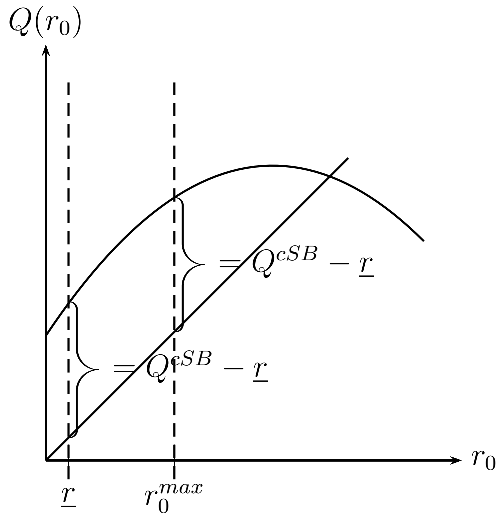


Figure 2: Fulfillment and violation of the existence condition.



(a) Principal's payoff for $r_t \in [\underline{r}, r_0]$.



(b) Range of self-enforcing efficiency wages $[\underline{r}, r_0^{max}]$.

Figure 3

5.3 The Optimal Incentive Contract

Based on the insights of the preceding analysis, the next result summarizes the key properties of the optimal incentive contract for an employed job seeker:

Proposition 3. *The optimal incentive contract, denoted by* $(\{b_t^*\}_{t=1}^{\infty}, r_0^*)$, *...*

(i) ... is stationary i.e. $(b_t^*, r_0^*) = (b^*, r_0^*)$ *for all* t *with* $d_t = 1$,

(ii) ... pays an efficiency wage, i.e. $r_0^* = r_0^{max} > \underline{r}$ *iff* (EW) *holds,*

(iii) ... features excessively steep performance incentives,

(iv) ... induces overworking, i.e. $e^* + s^* > e^{FB} + s^{FB}$,

(v) ... does not maximize the joint surplus, i.e. $Q^* < Q^{FB}$.

The following paragraph summarizes the economic forces that determine the shape of the optimal incentive contract. Under (EW), the rent-extraction-efficiency tradeoff has an interior solution. Consequently, in that case, the optimal incentive contract pays an efficiency wage. However, since the agent will never receive the full employment rent, he has excessive incentives to search, which in turn makes it optimal for the principal to raise the bonus above its first-best level. Thus, when the agent conducts OJS, it is optimal to combine efficiency wages and bonus pay because they influence the agent's incentives via different channels. This contrasts with the classic shirking model in which bonus pay and efficiency wages are never used in conjunction because they are just two different ways of rewarding the agent for good performance. Instead, in the present model, efficiency wages are not paid to reward good performance. Instead, they reward the agent for sampling low outside options. Moreover, in contrast to existing principal-agent models with limited liability in which the principal faces a similar tradeoff, she addresses the moral-hazard problem with an over- rather than an underprovision of performance incentives. By doing so, she shifts the agent's focus from search to work effort. Furthermore, under the optimal incentive contract, $Q_e < 0$ and $Q_s < 0$, which implies $e^* + s^* > e^{FB} + s^{FB}$. Hence, when the agent conducts OJS, the optimal incentive contract induces overworking.

Corner Solutions Proposition 3 characterizes the optimal incentive contract for active job seekers. The next result states under what conditions the optimal incentive contract implements only work, but no search effort.

Proposition 4. *The optimal incentive contract implements only work effort* $(e^* > s^* = 0)$ *if and only if*

$$f'(e^{FB})\Delta y > \frac{\delta}{1-\delta}\frac{\partial E_r\left(r_{t+1}|s=0\right)}{\partial s_t}. \tag{6}$$

Under (6), $(b^*, r_0^*) = (\Delta y, \underline{r})$ *and* $(e^*, s^*) = (e^{FB}, s^{FB})$, *with* $s^{FB} = 0$.

Under (6), search effort has a very weak impact on the distribution of r. Thus, Proposition 4 states that when the marginal effect of search effort is sufficiently small, the agent is discouraged from conducting any OJS at all, even if he does not receive any rent. In that case, the principal is able to implement first-best effort incentives and simultaneously extract all gains from trade.

Empirical Implications Propositions 3 and 4 imply that, everything else equal, performance pay and pay levels should be higher when OJS is more effective in generating outside offers for the worker. During the last twenty to thirty years, workers' search technology has improved dramatically as advancements in information and communication technology have considerably facilitated the search process. The comparison of Propositions 3 and 4 suggests that this development may have contributed to the surge in performance pay and pay levels over the last decades by raising worker's incentives to conduct OJS.

Furthermore, OJS will also be more effective when a worker's services are highly valued by other employers in the market.[25] Recent years have seen an intensification in competition among firms for the best and most highly talented workers in the labor market. Frydman (2007) suggests that this increase in competition reflects a shift in firms' demand from specific to more general human capital such as managerial skills. In light of this evidence, the results from this section suggest that increased competition for talented workers has also contributed to the change in the shape of compensation. In general, the paper's results suggest that the observed increase in the size of paid performance bonuses and salaries reflects an optimal adjustment of firms' compensation policies to increased search activities by their employees. However, while increased labor market mobility and increased competition among firms has been typically praised as improving the matching process in the labor market, this paper suggest a flip side. To the extent that higher labor market mobility and competition raises workers' incentives to perform OJS, it also generates a distortion of workers' effort incentives, as employed job seekers put an excessive emphasis on search effort and overwork.

6 Renegotiation

A common concern in the literature on repeated games is that if one assumes that players agree on playing a certain equilibrium at the start of the game, then they should also be able to renegotiate continuation play at any later moment in the game. In this section, I characterize the optimal incentive contract under the assumption that players are free to renegotiate continuation play at any decision node. Therefore, in addition to being a PPE, I now also require the optimal incentive contract to be renegotiation-proof. In particular, I require that it must satisfy the criterion of *Weak Renegotiation Proofness* (WRP) introduced by Farrell and Maskin (1989). A PPE is weakly renegotiation-proof if it does not have two continuation equilibria of which one strictly Pareto-dominates the other. WRP implies the following result:

Lemma 6. *An optimal incentive contract that pays an efficiency wage ($r_0^* > \underline{r}$) is weakly renegotiation-proof if and only if the efficiency wage is paid from period $t = 1$ onward ($u_1(r_1, 1) = r_0^*$ if $r_1 < r_0^*$).*

To understand the economic forces behind Lemma 6, consider a PPE that pays r_0 only from date 2 onward. When players can renegotiate continuation play, the agent would always

[25]Conditional on search effort, better outside options imply a shift of mass from low to high outside options.

tolerate a defection by the principal at some date $t > 1$ if it is coupled with the promise to pay r_0 from $t+1$ on. The reason is that the continuation equilibrium in which the principal withholds a promised rent at some $t > 1$ but promises to pay it from $t+1$ onward, is identical to the continuation equilibrium of the original PPE that starts at $t = 1$. Further, the original PPE strictly Pareto-dominates the continuation equilibrium that would punish the principal's deviation, as the agent gets his reservation payoff in either PPE, whereas by (EC), the principal gets a strictly higher payoff under the original PPE. Hence, PPE in which the principal promises to pay r_0 from $t \geq 2$ on do not satisfy WRP.

By contrast, when the efficiency wage has to be paid from the very first period on, a renegotiation off the equilibrium path will not occur. To see this, note that such a PPE has no continuation equilibrium in which the agent accepts a spot contract at some date t that yields him a payoff $r_t < r_0$, but under which the agent chooses (e_t, s_t) as if he would expect to receive r_0 from the next period on.

Lemma 6 further implies that the optimal incentive contract under renegotiation depends on the agent's initial-period outside option. To see this, note that at $t = 1$, the principal must always pay the agent at least r_1 in order to ensure his participation. However, if the principal prefers to implement some wage floor $r_0 > r_1$, she must pay the agent r_0. In particular, define $\widehat{r}_0 \equiv \arg\max_{r_0} Q(r_0) - r_0$ and let $\rho(r_0)$ denote the equilibrium separation threshold under an efficiency wage of r_0. When $\widehat{r}_0 > r_1$, it is profitable for the principal to raise $u_1(r_1, 1)$ to $\widehat{r}_0$, because this allows her to implement a wage floor of $r_0 = \widehat{r}_0$ which is by definition more profitable than $r_0 = r_1$. By contrast, when $\widehat{r}_0 \leq r_1$, implementing some $r_0 > r_1$ by raising the agent's payoff $u_1(r_1, 1)$ above his outside option r_1 is not profitable. However, since (PC) dictates that r_1 be paid anyways and because $Q'(r_0) > 0$, it is profitable for the principal to implement r_1 (if possible) as the wage floor. The next result summarizes:

Lemma 7. *The optimal efficiency wage r_0^* featured under an optimal incentive contract that satisfies WRP is given by:*

$$r_0^* = \begin{cases} \widehat{r}_0, & \text{if } r_1 \leq \widehat{r}_0, \\ r_1, & \text{if } \widehat{r}_0 < r_1 \leq r_0^{max}, \\ r_0^{max}, & \text{if } r_0^{max} < r_1 \leq \rho(r_0^{max}). \end{cases}$$

Figure 3 illustrates the optimal efficiency wage under renegotiation as a function of r_1.

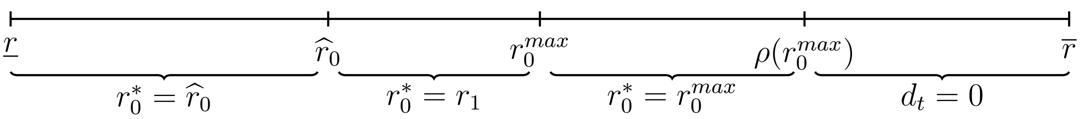


Figure 3: Optimal efficiency wage as a function of r_1

Employment Dynamics Lemma 7 implies that under renegotiation, the contracting parties may have an incentive to change the efficiency wage along the equilibrium path. To understand this, note first that the agent always prefers a higher wage floor. Further, Lemma 7 implies that the principal prefers to raise the wage floor whenever the agent's outside option exceeds the wage floor that is currently in place. Hence, at any date t with $r_t > r_0$ and $r_0 < r^{max}$, the contracting parties would find it profitable to renegotiate the wage floor from r_0 to r_t. It follows that, under renegotiation, the efficiency wage will sometimes be adjusted upward but never downward. Thus, renegotiation generates endogenous wage rigidity. Further, a higher efficiency wage also raises the joint surplus and reduces the agent's search effort, which in turn lowers the separation threshold. Hence, under renegotiation, the optimal incentive contract gives rise to non-trivia dynamics that are highly consistent with empirical evidence on wage and turnover dynamics in employment relationships (see e.g. Farber (1999) and Topel and Ward (1992)):

Proposition 5. *Under an optimal incentive contract that satisfies WRP, ...*

(i) ... the agent's expected payoff (weakly) increases with tenure.

(ii) ... the joint surplus (weakly) increases with tenure.

(iii) ... the separation rate (weakly) decreases with tenure.

7 Applications of The Framework

7.1 Unverifiable Output

Throughout the preceding analysis, it was assumed that output y is verifiable, which implied that the payment of the bonus rate could be enforced. However, in many jobs, especially for high-skilled workers, performance is notoriously difficult to assess objectively, let alone verify to third parties. A growing literature studies how the incentive problems that result in such incomplete contracting settings can be mitigated with the help of relational incentive contracts that make the payment of performance bonuses self-enforcing.[26] In this section, I derive the optimal relational incentive contract that the principal offers when the agent can exert search effort and output is unverifiable.

Thus, suppose now that while y is observable by the principal and the agent, it cannot be verified to third parties. In that case, the principal offers the agent a relational incentive contract under which she promises to pay not only the wage floor r_0 whenever $r_0 > r_t$, but also to pay the bonus b whenever $y_t = \overline{y}$ (and $d_t = 1$). Further, suppose that conditional on accepting this implicit contract, the agent terminates the relationship whenever the principal fails to pay b when $y_t = \overline{y}$, i.e. his strategy prescribes $d_t = 0$ if $y_{t-1} = \overline{y}$ and $b_{t-1} = 0$. [27]

[26] See Section 2 for references.

[27] Terminating the relationship in response to a deviation from the implicit agreement is a standard assumption in the relational-contracting literature. Since players cannot do worse than receive their current outside

Without loss of generality, suppose that the principal seeks to implement a stationary effort profile (e,s) and hence offers the stationary relational incentive contract (b, r_0).[28] I begin as in Section 5 with the case in which the agent receives no rent, i.e. $u_t = r_t$ for all t. The requirement that the bonus payment be self-enforcing adds another constraint to the principal's problem. To identify the levels of b that are self-enforcing, consider the principal's decision problem at the end of some period t with $y_t = \overline{y}$. Conditional on the separation threshold ρ, if she pays the bonus b as promised, her continuation payoff is

$$-(1-\delta)b + \delta \left\{ G(\rho|s) \left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) \right] + \left[1 - G(\rho|s)\right] \overline{\pi} \right\}. \tag{7}$$

If she withholds the bonus, the agent severs the relationship which implies a continuation payoff of $\delta\overline{\pi}$. Thus, a stationary bonus payment b that implements effort profile (e,s) is self-enforcing if and only if $\delta\overline{\pi}$ is no greater than (7). This leads to the following *dynamic enforcement constraint*, which states that any self-enforcing bonus payment must satisfy:

$$(1-\delta)b \leq \delta G(\rho|s) \left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) - \overline{\pi} \right]. \tag{DE}$$

Defining $b^{max}(e,s)$ as the maximum bonus payment that is self-enforcing under effort profile (e,s), (DE) can be rewritten as

$$b \leq b^{max}(e,s) \equiv \frac{\delta}{1-\delta} G(\rho|s) \left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) - \overline{\pi} \right].$$

The maximum bonus that the principal can credibly promise to pay is equal to the expected discounted rent that she receives under the relationship multiplied by the retention rate. If $b^{NR} \leq b^{max}(e^{NR}, s^{NR})$, the optimal incentive contract under verifiable output can also be implemented when output is unverifiable. I will thus focus on the case $b^{NR} > b^{max}(e^{NR}, s^{NR})$, in which the optimal bonus payment cannot be supported by a relational incentive contract. Let $(e(b), s(b))$ denote the agent's optimal effort choice if the bonus payment is b (and $r_0 = \underline{r}$) and let $Q(b)$ denote the corresponding joint surplus. By construction of b^{NR}, the principal's payoff is increasing in b for any $b < b^{NR}$. Hence, when $b^{NR} > b^{max}(e^{NR}, s^{NR})$, the optimal bonus payment under $r_0 = \underline{r}$ is the highest bonus that satisfies (DE). Denote this bonus by $\widetilde{b}^{NR}$:

$$\widetilde{b}^{NR} = \frac{\delta}{1-\delta} G(\widetilde{\rho}^{NR}|s(\widetilde{b}^{NR})) \left[Q(\widetilde{b}^{NR}) - \mathbb{E}\left(r|r \leq \widetilde{\rho}^{NR}, s(\widetilde{b}^{NR})\right) - \overline{\pi} \right],$$

Further, $Q(\widetilde{b}^{NR}) < Q^{NR}$ and $\widetilde{\rho}^{NR} < \rho^{NR}$. Thus, if $b^{NR} > b^{max}(e^{NR}, s^{NR})$, the implemented surplus is lower than Q^{NR}.

Next, consider the effect of implementing a wage floor. When $b^{NR} > b^{max}(e^{NR}, s^{NR})$, the principal would like to raise b above $\widetilde{b}^{NR}$, but (DE) prevents her from doing so.[29] Hence, at

options ad infinitum, such a response to a deviation by the principal constitutes an optimal punishment in the sense of Abreu, Pearce, and Stacchetti (1990).

[28]For the same arguments as in the main part of the analysis, attention can be restricted to stationary contracts.

[29]When $b^{NR} \leq b^{max}(e^{NR}, s^{NR})$, the effect of efficiency wages is the same as in Section 5.2.

$(b, r_0) = (\widetilde{b}^{NR}, \underline{r})$:

$$\frac{\partial Q(e,s)}{\partial b} = Q_e \frac{\partial e}{\partial b} + Q_s \frac{\partial s}{\partial b} > 0,$$

and by Corollary 1:

$$\frac{\partial Q(e,s)}{\partial r_0} = Q_e \frac{\partial e}{\partial r_0} + Q_s \frac{\partial s}{\partial r_0} > 0.$$

Thus, as before, a higher r_0 raises the joint surplus. Further, when output is unverifiable, raising the wage floor has an additional effect. If (DE) is binding, raising r_0 relaxes (DE), thereby enabling the principal to provide higher-powered incentives via the bonus scheme. To see this, note that r_0 raises b^{max} only if the associated gain in surplus outweighs the increase in the principal's wage bill. Yet, when $r_0 > \underline{r}$, the agent earns a rent from employment, which can be used to induce him to pay a fine $-\underline{b} > 0$ to the principal whenever $y_t = \underline{y}$. Thus, consider the relational contract that prescribes that the principal pays $\overline{b} > 0$ to the agent if $y_t = \overline{y}$ and that the agent pays $-\underline{b} > 0$ to the principal when $y_t = \underline{y}$. Further, suppose that the principal terminates the relationship if the agent fails to pay the fine when $y_t = \underline{y}$. Complying with this relational contract when $y_t = \underline{y}$ yields the agent a continuation payoff of

$$(1-\delta)\underline{b} + \delta \left[G(r_0|s) r_0 + \int_{r_0}^{\overline{r}} r g(r|s) dr \right].$$

Instead, refusing to pay the fine and inducing severance implies a payoff of

$$\delta \int_{\underline{r}}^{\overline{r}} r g(r|s) dr.$$

Hence, the payment of the fine is self-enforcing if and only if

$$-\underline{b} \leq \frac{\delta}{1-\delta} \int_{\underline{r}}^{r_0} (r_0 - r)\, g(r|s) dr.$$

Let $-\underline{b}^{min}(s)$ denote the maximum fine the agent can be induced to pay given search effort level s:

$$-\underline{b}^{min}(s) \equiv \frac{\delta}{1-\delta} \int_{\underline{r}}^{r_0} (r_0 - r)\, g(r|s) dr \geq 0.$$

Next, define the *effective bonus rate* as $\Delta b \equiv \overline{b} - \underline{b}$. Analogously to (2), we have:

$$\frac{\partial e_t^A}{\partial \Delta b} > 0, \frac{\partial s_t^A}{\partial \Delta b} \leq 0 \text{ and } \frac{\partial e^A + s^A}{\partial \Delta b} > 0. \tag{8}$$

Given r_0, the highest effective bonus rate that can be sustained under any relational contract that implements (e, s) is $\Delta b^{max}(e,s) \equiv b^{max}(e,s) - \underline{b}^{min}(s)$.

Lemma 8. *For every (e, s), $\Delta b^{max}(e,s)$ is strictly increasing in $r_0 \in \left[\overline{r}, \rho^{FB}\right]$.*

Thus, when output is unverifiable, efficiency wages improve the principal's ability to provide effort incentives via the bonus scheme. Taking this effect into account, the overall impact of introducing an efficiency wage on the joint surplus at $r_0 = \underline{r}$ is:

$$\frac{\partial Q(e,s)}{\partial r_0}|_{(\Delta b, r_0)=(\widetilde{b}^{NR},\underline{r})} = Q_e \frac{\partial e}{\partial r_0} + Q_s \frac{\partial s}{\partial r_0} + \frac{\partial Q(e,s)}{\partial b}\frac{\partial b}{\partial r_0}. \tag{9}$$

By Lemma 8, when (DE) binds, the last term in this expression is strictly positive. Let $(\Delta\widetilde{b}^*, \widetilde{r}_0^*)$ denote the optimal incentive contract when output is unverifiable.

Proposition 6. *The optimal incentive contract under unverifiable output,* $(\Delta\widetilde{b}^*, \widetilde{r}_0^*)$ *is characterized by* $\Delta\widetilde{b}^* \leq b^*$ *and* $\widetilde{r}_0^* \geq r_0^*$. *Further, these inequalities are strict if* (EW) *holds and* (DE) *binds.*

Thus, when output is unverifiable, efficiency wages not only improve the agent's effort incentives, they also raise the size of the maximum discretionary bonus that the principal can credibly promise to pay. Hence, efficiency wages are more likely to be part of an optimal incentive contract as compared to when the bonus can be legally enforced.

7.2 Limited Liability

In the main part of the analysis, the assumption that the fixed wage could become negative played a crucial role for the derivation of the optimal incentive contract. Since $b^* \geq \Delta y$, the optimal incentive contract would always specify a negative fixed-wage component. However, in many jobs, such negative wages are not feasible, e.g. because of a minimum wage requirement or because the employees lack the financial resources to cover such negative wages. In this section, I address these situations by deriving the optimal incentive contract under OJS when the agent is protected by limited liability, i.e., I analyze the model under the restriction

$$W_t = w_t + b_t \geq 0 \text{ for any } t. \tag{LL}$$

As a first step, it is useful to start with the case in which the principal offers the same contract $(0, b)$ in every period t with $d_t = 1$. As such a contract induces a stationary effort profile, the principal's payoff at any date t with $d_t = 1$, given separation threshold ρ is:

$$\pi = (1-\delta)\left[\underline{y} + f(e)(\Delta y - b)\right] + \delta\left(G(\rho|s)\pi + [1 - G(\rho|s)]\,\overline{\pi}\right),$$

Since by (PC), $\pi \geq \overline{\pi}$, this expression indicates that under limited liability, the principal derives no value from a positive search level by the agent. In particular, the prohibition of negative fixed wages prevents the principal from extracting the agent's discounted benefits from search effort. The agent's payoff given $(0, b)$ and ρ is:

$$u = (1-\delta)\left[f(e)b - c(e+s)\right] + \delta\left[G(\rho|s)u + \int_{\rho}^{\overline{r}} rg(r|s)dr\right].$$

Since the agent's payoff u is the same in every period in which he accepts the principal's offer, the equilibrium separation threshold is just equal to the agent's equilibrium payoff, i.e. $u = \rho$.

Thus, under the stationary contract $(0, b)$, the agent earns a limited-liability rent $\rho - r_t$ at any date t with $d_t = 1$ and the principal can no longer choose b and $r_0 = \rho$ independently. Rather, since $u = \rho$, ρ is determined by the agent's optimal effort choice, given $(0, b)$. The agent chooses (e, s) to maximize:

$$\rho = \frac{1-\delta}{1-\delta G(\rho|s)}\left[f(e)b - c(e+s)\right] + \frac{\delta}{1-\delta G(\rho|s)}\int_{\rho}^{\overline{r}} rg(r|s)dr.$$

Using the implicit function theorem, the first-order conditions characterizing the agent's choice of (e, s) are

$$f'(e)b = c'(e+s),$$

$$\frac{\delta}{1-\delta}\left[G_s(\rho|s)\rho + \int_{\rho}^{\overline{r}} rg_s(r|s)dr\right] = c'(e+s).$$

Clearly, under the stationary contract $(0, b)$, the agent's returns from search effort are determined by ρ. Let $\rho(b)$ be the agent's equilibrium payoff when $d_t = 1$ as a function of the stationary contract $(0, b)$. By the envelope theorem:

$$\frac{\partial\rho(b)}{\partial b} = \frac{1-\delta}{1-\delta G\left(\rho(b)|s\right)} f(e) > 0.$$

Thus, under (LL), a higher bonus rate does not only raise the returns from work effort, it also raises the agent's equilibrium payoff, thereby reducing the returns from search effort.

The principal thus chooses b to maximize π subject to the agent's incentive compatibility constraints and $\rho(b)$. Let $b^{\dagger}$ denote the solution to the principal's problem and let $\pi^{\dagger}$ and $\rho^{\dagger}$ denote the implied payoff levels. Under the stationary contract $(0, b)$, $\pi^{\dagger}$ and $\rho^{\dagger}$ are the same in every period. But then, if $\pi^{\dagger} > \overline{\pi}$, there are unexploited gains from trade. In particular, there exists $r_t > \rho$ such that $\pi^{\dagger} + \rho^{\dagger} \geq r_t + \overline{\pi}$ but under which no trade is conducted. For these realizations of r_t, the principal can raise her payoff by offering the agent a contract that yields him exactly r_t. It follows that the optimal incentive contract cannot specify the same pair $(0, b)$ in every period. The next result characterizes the optimal incentive contract under (LL):

Proposition 7. *When the agent is protected by limited liability (i.e., when (LL) holds), the optimal incentive contract specifies a threshold r_0^{ll} such that ...*

(i) ... for all t with $r_t \leq r_0^{ll}$, the optimal incentive contract stipulates a constant bonus b_0^{ll}, inducing a constant effort profile (e_0^{ll}, s_0^{ll}), and yielding principal and agent constant payoffs of π_0^{ll} and r_0^{ll} respectively.

(ii) ... for all t with $r_0^{ll} < r_t \leq \rho^{ll}$, the optimal incentive contract stipulates a bonus $b^{ll}(r_t)$ that is strictly increasing in r_t and that induces an effort profile $(e^{ll}(r_t), s^{ll}(r_t))$ with e^{ll} (s^{ll}) being strictly increasing (decreasing) in r_t. Further, the principal receives a payoff $\pi^{ll}(r_t)$ that is strictly decreasing in r_t, while the agent receives r_t.

(iii) ... the equilibrium separation threshold ρ^{ll} is defined by $\pi^{ll}(\rho^{ll}) = \overline{\pi}$.

Thus, when the agent is protected by limited liability, the optimal incentive contract is non-stationary as the optimal bonus and the agent's behavior change with r_t. Further, the rent that the agent receives under (LL) is not a pure limited-liability rent. In particular, an increase in b affects the agent's incentives via two channels. First, it raises the marginal returns of work effort and second, it increases r_0^{ll} which reduces the agent's marginal returns from search. The latter effect is absent in standard moral-hazard models without search effort. Therefore, under OJS, the agent's rent exceeds the standard limited liability rent and, since $w_t = 0$, the optimal bonus is also larger than the second-best bonus in a standard agency model with limited liability but no search effort. In particular, optimal incentive contract stipulates $w_t = 0$ for all t, because when $r_t > r_0^{ll}$, it is more profitable for the principal to raise the agent's payoff via b instead of w. Intuitively, while an increase in w represents a mere transfer from the principal to the agent, increasing b raises the joint surplus. Thus, the principal can achieve a given increase of the agent's payoff more cheaply by raising b than by raising w.

7.3 Different Cost Functions

Throughout the paper, it was assumed that the agent's effort costs are increasing in the sum of work and search effort. However, the shape of the optimal incentive contract, characterized by Proposition 3 does not require work and search effort to be *perfect* substitutes in the agent's cost function. In particular, suppose instead that the agent's effort costs are given by the function $c(e, s)$, which is increasing and convex in each of its arguments.

Proposition 8. *For a general cost function $c(e, s)$, the optimal incentive contract is characterized by Proposition 3 if and only if $c_{es} = c_{se} > 0$.*

Intuitively, as long as $c_{es} = c_{se} > 0$, the agent faces an effort-substitution problem which implies that s^A is decreasing in b_t thus justifying the use of excessive bonus pay. Furthermore, the rent-seeking motive of OJS is always present, which implies that a larger wage floor raises the joint surplus.

By contrast, if work and search effort are technologically independent, i.e. $c_{es} = c_{se} = 0$, the optimal incentive contract does not feature excessive performance pay, but does pay an efficiency wage. Intuitively, when $c_{es} = c_{se} = 0$, performance pay no longer plays a dual role since the agent's search activities are independent of b. Hence, the optimal bonus is equal to its first-best level. However, paying an efficiency wage is still profitable. In fact, it is even more profitable, because it is not associated with an undesirable increase in work effort.

7.4 The Case of the Most Productive Job

Throughout the paper, it was assumed there are other jobs in the economy where the agent is strictly more productive. In fact, this assumption is rather weak, as it applies to every firm except the one that has the highest valuation for the worker's services. In this subsection, I drop assumption A1 and instead study the optimal incentive contract under OJS for the agent's best job match in the labor market. In particular, I study a principal-agent relationship for which it is never optimal to separate under the optimal incentive contract.

Assumption A2.

$$Q^* > \overline{r} + \overline{\pi}.$$

Under A2, search effort entails no social benefits. Hence, the first-best level of search effort is zero. To make the problem interesting, I assume that (6) does not hold, which implies that under full rent extraction, the agent will provide a positive level of search effort. However, under A2, the agent's incentives to search are exclusively driven by his rent-seeking motive. It follows that first-best search effort incentives require that the agent's payoff is at least as large as his maximal outside option, $\overline{r}$. In turn, because $Q^* - \overline{\pi} > \overline{r}$, A2 implies that the principal does not have to concede the full employment rent to the agent to implement the first-best.

Proposition 9. *Under A2, if an employment relationship can be sustained when the principal extracts all gains from trade, then there exists a PPE that implements the first-best if and only if:*

$$Q^{NR} - \underline{r} \leq Q^{FB} - \overline{r}.$$

Proposition 9 indicates that under A2, the conditions under which the first-best can be supported as part of a PPE are weaker than under A1. However, besides A2 increasing the scope for implementing efficient effort incentives, A2 does not change the paper's main results.

7.5 Long-Term Contracting

A key driver behind the paper's results is that the principal cannot commit to a long-term contract. If the principal could commit to any sequence of spot contracts, she would be able to implement first-best effort incentives while extracting all the rents from the relationship up-front. To see this, recall that under spot contracting, the payment of an efficiency wage has to be self-enforcing, which limits the maximum efficiency wage that the principal can credibly promise to pay. By contrast, if the principal can commit to a long-term contract, she can implement any wage floor she likes. Further, when the principal has commitment power, there is no need for her to pay the wage floor already in the first period, because the long-term contract insures the agent against possible defections by the principal in future periods. The next result characterizes the optimal long-term contract under commitment.

Proposition 10. *If the principal can commit to any sequence* $\{(w_t, b_t)\}_{t=1}^{\infty}$*, then the optimal long-term contract* $\left\{(w_t^{LT}, b_t^{LT})\right\}_{t=1}^{\infty}$ *implements first-best effort incentives and extracts all rents from the agent in the first period of interaction. In particular, the optimal long-term contract stipulates* $b_t^{LT} = \Delta y$ *for all* t*,* $u_1 = r_1$ *and* $r_t = \rho^{FB}$ *for all* $t \geq 2$*.*

Thus, under full commitment, the principal essentially sells the firm to the agent at date $t = 1$ for a price of $Q^{FB} - r_1$. While long-term contracting eliminates the distortion caused by OJS, spot contracting is arguably the more realistic assumption. First, spot contracting is the appropriate assumption for jobs where employment is at will. Further, even when employment is not at-will, many factors that contribute to the value that a worker attaches to his job are informal promises and perks that, by their very nature, cannot be enforced by the courts. The provision of these benefits in long-term contracts must therefore also be self-enforcing.

8 Conclusion

One of the key objectives of incentive theory is to explain the shape of compensation contracts observed in real-life employment relationships. This paper has investigated how the shape of optimal incentive contracts is affected by workers' incentives to prompt competition for their services through on-the-job search (OJS). In light of the large incidence of OJS and employer-to-employer transitions in today's labor market, this is a very pressing issue for many employers. I have shown that under OJS by the agent, the principal's optimization problem involves a tradeoff between rent-extraction and efficiency. Further, the shape of the optimal incentive contract differs markedly from the corresponding contract for non-searching workers. Instead of an underprovision, the optimal incentive contract entails an overprovision of performance pay. Moreover, the principal deliberately pays the agent a rent in addition to these excessive bonuses. Further, when allowing for renegotiation, the model predicts that average wages and the agent's productivity increase with tenure, while separation rates decrease with tenure. The paper's results are highly consistent with empirical observations. First, general levels of pay and in particular performance pay have considerably increased in many professions over recent decades. At the same time, OJS has become an increasingly common practice. This paper suggests a link between these two developments. Moreover, the paper also offers an alternative perspective on the use of efficiency wages in incentive contracts. Under OJS, efficiency wages are complementary to bonus pay and need not be tied to a threat of dismissal to provide effort incentives.

Appendix

A Preliminaries

Proof of Lemma 1. Suppose contrary to the statement that there exist public histories such that $d_t = 1$ for every $r_t \in [\underline{r}, \overline{r}]$. By A1, the maximum joint surplus that can be generated in that case is $max_{(e,s)}T(e,s) + \delta(\overline{r} + \overline{\pi})$. But by A1, there exists $r' < \overline{r}$ such that for any $r > r'$, $r + \overline{\pi} > max_{(e,s)}T(e,s) + \delta(\overline{r} + \overline{\pi})$. Thus, if $r_t > r'$, (PC) cannot be satisfied, which contradicts the initial claim. □

Proof of Lemma 2. By Lemma 1, there is some $\rho_t < \overline{r}$ such that $d_t = 0$ whenever $r_t > \rho_t$. For any date t with $d_t = 1$, the principal's and agent's payoffs are respectively given by:

$$\pi_t = (1-\delta)\left[\underline{y} + f(e_t)\Delta y - w_t\right] + \delta \mathbb{E}_r\left[\pi_{t+1}|s_t\right]$$

$$u_t = (1-\delta)\left[w_t - c(e_t + s_t)\right] + \delta \mathbb{E}_r\left[u_{t+1}|s_t\right]$$

Joint surplus is thus given by

$$Q_t = (1-\delta)\left[\underline{y} + f(e_t)\Delta y - c(e_t + s_t)\right] + \delta \mathbb{E}_r\left[Q_{t+1}|s_t\right]$$

Observe that the joint surplus is independent of r_t. Let Q_t^{FB} be the maximum surplus that can be generated when $d_t = 1$ and let (e_t^{FB}, s_t^{FB}) denote the effort profile that implements it. Further, define $T_t^{FB} \equiv T(e_t^{FB}, s_t^{FB})$. Since Q_t is independent of r_t, joint-surplus maximization requires $Q_{t+1} = Q_t^{FB}$ whenever $d_{t+1} = 1$. From (PC), it follows that $\rho_{t+1} = \rho_t^{FB} = Q_t^{FB} - \overline{\pi}$ for all $r_{t+1} \in [\underline{r}, \overline{r}]$. Hence, joint surplus can be written as:

$$Q_t = Q_t^{FB} = (1-\delta)T_t^{FB} + \delta\left\{G(\rho_t^{FB}|s_t^{FB})Q^{FB} + \left[1 - G(\rho_t^{FB}|s_t^{FB})\right]\left[\overline{\pi} + \mathbb{E}_r\left(r_{t+1}|r_{t+1} > \rho_t^{FB}, s_t^{FB}\right)\right]\right\}$$

Rearranging this expression gives:

$$Q_t^{FB} = \frac{1-\delta}{1-\delta G(\rho_t^{FB}|s_t^{FB})}T^{FB} + \frac{\delta\left[1 - G(\rho_t^{FB}|s_t^{FB})\right]}{1-\delta G(\rho_t^{FB}|s_t^{FB})}\left[\overline{\pi} + \mathbb{E}_r\left(r_{t+1}|r_{t+1} > \rho_t^{FB}, s_t^{FB}\right)\right]. \tag{A.1}$$

Since this expression is independent of r_t, $(e_t^{FB}, s_t^{FB}) = (e^{FB}, s^{FB})$ and hence $Q_t^{FB} = Q^{FB}$ and $\rho^{FB} = \rho_t^{FB}$ for any $r_t \leq \rho^{FB}$ which completes the proof. □

First-Order Conditions under the First-Best. Since $\rho^{FB} = Q^{FB} - \overline{\pi}$, (A.1) defines Q^{FB} implicitly. Differentiating Q^{FB} with respect to e and s gives:

$$Q_e^{FB} = \frac{\partial Q^{FB}}{\partial e} + \frac{\partial Q^{FB}}{\partial \rho^{FB}}\frac{\partial \rho^{FB}}{\partial e} = 0, \tag{A.2}$$

$$Q_s^{FB} = \frac{\partial Q^{FB}}{\partial s} + \frac{\partial Q^{FB}}{\partial \rho^{FB}}\frac{\partial \rho^{FB}}{\partial s} = 0. \tag{A.3}$$

Define the implicit function:

$$R(e,s,\rho^{FB}) = Q^{FB} - \overline{\pi} - \rho^{FB} = 0.$$

Differentiating R with respect to ρ^{FB} gives after some algebra:

$$\frac{\partial R(e,s,\rho^{FB})}{\partial \rho^{FB}} = \frac{\partial Q^{FB}}{\partial \rho^{FB}} - 1 = -\left[1 - G(\rho^{FB}|s)\right]. \tag{A.4}$$

By the implicit function theorem:

$$\frac{\partial \rho^{FB}}{\partial e} = -\frac{\frac{\partial R(e,s,\rho^{FB})}{\partial e}}{\frac{\partial R(e,s,\rho^{FB})}{\partial \rho^{FB}}} = -\frac{\partial Q^{FB}}{\partial e}\frac{-1}{1-G(\rho^{FB}|s)}, \tag{A.5}$$

$$\frac{\partial \rho^{FB}}{\partial s} = -\frac{\frac{\partial R(e,s,\rho^{FB})}{\partial s}}{\frac{\partial R(e,s,\rho^{FB})}{\partial \rho^{FB}}} = -\frac{\partial Q^{FB}}{\partial s}\frac{-1}{1-G(\rho^{FB}|s)}. \tag{A.6}$$

Using (A.4), (A.5) and (A.6), (A.2) and (A.3) can be rewritten as:

$$Q_e^{FB} = \frac{\partial Q^{FB}}{\partial e}\frac{1}{1-G(\rho^{FB}|s)} = 0, \tag{A.7}$$

$$Q_s^{FB} = \frac{\partial Q^{FB}}{\partial s}\frac{1}{1-G(\rho^{FB}|s)} = 0. \tag{A.8}$$

Further, since $G(\rho^{FB}|s) < 1$ under A1, (A.7) and (A.8) are equivalent to:

$$\frac{\partial Q^{FB}}{\partial e} = 0,$$

$$\frac{\partial Q^{FB}}{\partial s} = 0,$$

which yields (FB_e) and (FB_s). □

Second-Order Conditions under the First-Best. Let

$$Q(e,s) = \frac{1-\delta}{1-\delta G(\rho|s)}T(e,s) + \frac{\delta\left[1-G(\rho|s)\right]}{1-\delta G(\rho|s)}\left[\overline{\pi} + \mathbb{E}_r\left(r|r>\rho,s\right)\right]. \tag{A.9}$$

To economize on notation, I drop the ^{FB}-superscripts in what follows, with the understanding that all derivations that follow are evaluated at the first-best effort profile. The second-order derivatives of $Q(e,s)$ with respect to e and s respectively are:

$$Q_{ee} = \frac{1-\delta}{\left[1-\delta G(\rho|s)\right]^2}\left[f''(e)\Delta y - c''(e+s)\right] < 0. \tag{A.10}$$

$$\begin{aligned} Q_{ss} = &-\frac{1-\delta}{1-\delta G(\rho|s)}c''(e+s) \\ &+\frac{\delta}{1-\delta G(\rho|s)}\left[2G_s(\rho|s)\frac{\partial \mathbb{E}(r|r>\rho,s)}{\partial s} + G_{ss}(\rho|s)\mathbb{E}(r|r>\rho,s) + \int_{\rho}^{\overline{r}} r g_{ss}(r|s)dr\right] \\ &-\frac{2\delta(1-\delta)G_s(\rho|s)}{\left[1-\delta G(\rho|s)\right]^2}\left[c'(e+s) + \frac{\partial \mathbb{E}(r|r>\rho,s)}{\partial s}\right] \\ &+\frac{\delta(1-\delta)}{\left[1-\delta G(\rho|s)\right]^3}\left[\left(1-\delta G(\rho|s)\right)G_{ss}(\rho|s) + 2\delta\left[G_s(\rho|s)\right]^2\right]\left[T(e,s) - \mathbb{E}(r|r>\rho,s) - \overline{\pi}\right]. \end{aligned} \tag{A.11}$$

While the first, second and fourth term of Q_{ss} are unambiguously negative, the third term is positive. As s increases, the weight attached to $T(e,s)$ and hence to the marginal costs of effort $c'(e+s)$ decreases, while the weight attached to the payoff under separation and hence the marginal benefit of increasing that payoff increase. For Q_{ss} to be negative, this effect must be sufficiently small relative to the effects captured by terms 1, 2 and 4.

Using (FB_s) and collecting terms, (A.11) can be simplified to:

$$\begin{aligned} Q_{ss} &= \frac{-(1-\delta)}{1-\delta G(\rho|s)} c''(e+s) \\ &+ \frac{\delta}{1-\delta G(\rho|s)} \left[G_{ss}(\rho|s)\mathbb{E}(r|r>\rho,s) + \int_{\rho}^{\overline{r}} r g_{ss}(r|s)dr \right] \\ &+ \frac{\delta(1-\delta)}{[1-\delta G(\rho|s)]^2} \left[(1-\delta G(\rho^*|s)) G_{ss}(\rho|s) \right] [T(e,s) - \mathbb{E}(r|r>\rho,s) - \overline{\pi}]. \end{aligned} \tag{A.12}$$

The first and the third are unambiguously negative. The second term is negative if

$$E(r|r>\rho,s) < -\int_{\rho}^{\overline{r}} r \frac{g_{ss}(r|s)}{G_{ss}(\rho|s)} dr \text{ for } \rho < \overline{r}. \tag{A.13}$$

Condition (A.13) essentially requires $G(.|.)$ to be sufficiently convex. Note, however that it is a sufficient condition. $Q_{ss} < 0$ will generally be satisfied for weaker restrictions, e.g. if $c(.)$ is sufficiently convex.

Further, the cross-partial derivative is given by:

$$Q_{es} = -\frac{1-\delta}{[1-\delta G(\rho|s)]^2} c''(e+s) < 0. \tag{A.14}$$

Under condition (A.13), all terms in Q_{ee} and Q_{ss} are negative. Therefore, $Q_{ee}Q_{ss} - Q_{es}^2 > 0$ implying that the solution is the unique maximum. □

The Agent's Decision Problem. Differentiating the agent's objective function in (IC) with respect to e_t and s_t gives:

$$f'(e_t)b_t - c'(e_t + s_t) = 0,$$

$$\frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u_{t+1}(r, d_{t+1}) g_s(r|s) - c'(e_t + s_t) = 0.$$

Concavity of $f(.)$ and convexity of $c(.)$ and $G(.|.)$ ensure that these two equations define the unique global maximum (e_t^A, s_t^A). The marginal effects of a change in b_t on e_t^A and s_t^A can be obtained using the implicit function theorem. In particular, we have:

$$\frac{\partial e_t^A}{\partial b} = \frac{-f'(e_t) \left[\frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u(r, d_{t+1}) g_{ss}(r|s_t) dr - c''(e_t + s_t) \right]}{[f''(e_t)b - c''(e_t + s_t)] \frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u(r, d_{t+1}) g_{ss}(r|s_t) dr - f''(e_t) \cdot b \cdot c''(e_t + s_t)} > 0.$$

$$\frac{\partial s_t^A}{\partial b} = \frac{-f'(e_t) c''(e_t + s_t)}{[f''(e_t)b - c''(e_t + s_t)] \frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u(r, d_{t+1}) g_{ss}(r|s_t) dr - f''(e_t) \cdot b \cdot c''(e_t + s_t)} < 0.$$

$$\frac{\partial (e_t^A + s_t^A)}{\partial b} = \frac{-f'(e_t) \left[\frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u(r, d_{t+1}) g_{ss}(r|s_t) dr \right]}{[f''(e_t)b - c''(e_t + s_t)] \frac{\delta}{1-\delta} \int_{\underline{r}}^{\overline{r}} u(r, d_{t+1}) g_{ss}(r|s_t) dr - f''(e_t) \cdot b \cdot c''(e_t + s_t)} > 0.$$

The signs of the respective marginal effects follow from the assumptions on $f(.)$, $c(.)$ and $G(.|.)$. □

Proof of Lemma 3. For any given effort profile, the right-hand sides of (FB_e) and (FB_s) are equal to the right-hand sides of (IC_e) and (IC_s). Hence, the agent chooses the first-best effort profile (e^{FB}, s^{FB}) if and only if:

$$f'(e)b_t = f'(e)\Delta y, \tag{A.15}$$

$$\frac{\partial E_r(u_{t+1}|s)}{\partial s} = G_s(\rho|s)\phi [T(e,s) - \mathbb{E}_r(r|r>\rho,s) - \overline{\pi}] + [1 - G(\rho|s)] \frac{\partial \mathbb{E}_r(r|r>\rho,s)}{\partial s}, \tag{A.16}$$

where (A.15) and (A.16) are evaluated at $(e,s) = (e^{FB}, s^{FB})$. (A.15) implies that first-best effort incentives require $b_t = \Delta y$ for all t. Further, the left-hand side of (A.16) can be rewritten as

$$\begin{aligned}
&\frac{\partial E_r\left(u_{t+1}|s\right)}{\partial s} \\
&= G_s(\rho^{FB}|s)\left[E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right) - E_r\left(r_{t+1}|r_{t+1} > \rho^{FB}, s\right)\right] \\
&+ G(\rho^{FB}|s)\frac{\partial E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right)}{\partial s} \\
&+ \left[1 - G(\rho^{FB}|s)\right]\frac{\partial E_r\left(r_{t+1}|r_{t+1} > \rho, s\right)}{\partial s}
\end{aligned} \tag{A.17}$$

Furthermore, (1) and $Q^{FB} - \equiv \rho^{FB}$ imply

$$\phi\left[T(e,s) - \mathbb{E}_r(r|r > \rho^{FB}, s) - \overline{\pi}\right] = \rho^{FB} - \mathbb{E}_r(r|r > \rho^{FB}, s). \tag{A.18}$$

Inserting (A.17) and (A.18) back into (A.16) and canceling terms yields:

$$\begin{aligned}
&G_s(\rho^{FB}|s)\left[\rho^{FB} - \mathbb{E}_r(r|r > \rho^{FB}, s)\right] \\
&= G_s(\rho^{FB}|s)\left[E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right) - E_r\left(r_{t+1}|r_{t+1} > \rho^{FB}, s\right)\right] \\
&+ G(\rho^{FB}|s)\frac{\partial E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right)}{\partial s}
\end{aligned}$$

Since $E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right) \leq \rho^{FB}$, this equation holds if and only if $u_t = \rho^{FB}$ for any $r_t \leq \rho^{FB}$. In that case: $E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right) = \rho^{FB}$ and

$$\frac{\partial E_r\left(u_{t+1}|r \leq \rho^{FB}, s\right)}{\partial s} = 0$$

□

B Equilibrium Analysis

Fix $e_t = e$ and let $s_t^{NR}(e)$ denote the agent's optimal choice of search effort when $u_{t+1} = r_{t+1}$, given $e_t = e$ and let $s^{FB}(e)$ denote the first-best level of s given $e_t = e$.

Lemma 9. *For all t, we have $s_t^{NR}(e) > s^{FB}(e)$ for any level of e.*

Proof of Lemma 9. $s^{FB}(e)$ is implicitly defined by (FB_s):

$$\frac{\delta G_s(\rho^{FB}|s)}{1 - \delta G(\rho^{FB}|s)}\left[T(e,s) - \mathbb{E}_r\left(r|r > \rho^{FB}, s\right) - \overline{\pi}\right] + \frac{\delta}{1-\delta}\int_{\rho^{FB}}^{\overline{r}} r g_s(r|s) dr = c'(e+s) \tag{B.1}$$

Further, $s_t^{NR}(e)$ is implicitly defined by (IC_s^{NR}):

$$\frac{\delta}{1-\delta}\int_{\underline{r}}^{\overline{r}} r g_s(r|s) = c'(e+s). \tag{B.2}$$

The right-hand sides of these two equations are identical and increasing in e and s. Hence, $s_t^{NR}(e) > s^{FB}(e)$ if and only if the left-hand side of (B.2) exceeds the left-hand side of (B.1), which implies

$$\int_{\underline{r}}^{\overline{r}} r g_s(r|s) > \left\{G_s(\rho^{FB}|s)\phi\left[T(e,s) - \overline{\pi} - \mathbb{E}_r(r|r > \rho^{FB}, s)\right] + \left[1 - G(\rho^{FB}|s)\right]\frac{\partial \mathbb{E}_r\left(r|r > \rho^{FB}, s\right)}{\partial s}\right\} \tag{B.3}$$

The left-hand side of (B.3) is equivalent to

$$\left[1-G(\rho^{FB}|s)\right]\frac{\partial \mathbb{E}\left(r|r>\rho^{FB},s\right)}{\partial s}+G_s(\rho^{FB}|s)\left[\mathbb{E}\left(r|r\leq\rho^{FB},s\right)-\mathbb{E}\left(r|r>\rho^{FB},s\right)\right] \\ +G(\rho^{FB}|s)\frac{\partial \mathbb{E}\left(r|r\leq\rho^{FB},s\right)}{\partial s}. \tag{B.4}$$

Subtracting the right-hand side of (B.3) from (B.4) yields after some algebra:

$$G_s(\rho^{FB}|s)\left\{\phi\left[T(e,s)-\overline{\pi}\right]+(1-\phi)\,\mathbb{E}\left(r|r>\rho^{FB},s\right)-\mathbb{E}\left(r|r\leq\rho^{FB},s\right)\right\}-G(\rho^{FB}|s)\frac{\partial \mathbb{E}\left(r|r\leq\rho^{FB},s\right)}{\partial s}. \tag{B.5}$$

The second term of this expression is clearly negative. Further, $Q^{FB}=\rho^{FB}+\overline{\pi}$ and (1) imply

$$\phi\left[T(e,s)-\overline{\pi}\right]=\rho^{FB}-(1-\phi)\mathbb{E}\left(r|r>\rho^{FB},s\right) \tag{B.6}$$

Inserting (B.6) into the first part of (B.5) and canceling terms gives:

$$G_s(\rho^{FB}|s)\left[\rho^{FB}-\mathbb{E}\left(r|r\leq\rho^{FB},s\right)\right]. \tag{B.7}$$

This expression is negative as well, thus proving the initial claim. □

Proof of Lemma 4. By Lemma 1, there is some $\rho_t<\overline{r}$ such that $d_t=0$ whenever $r_t>\rho_t$. For any date t with $d_t=1$, the principal's and agent's payoffs are respectively given by:

$$\pi_t=(1-\delta)\left[\underline{y}+f(e_t)(\Delta y-b_t)-w_t\right]+\delta\mathbb{E}_r\left[\pi_{t+1}|s_t\right]$$

$$u_t=(1-\delta)\left[w_t+f(e_t)b_t-c(e_t+s_t)\right]+\delta\mathbb{E}_r\left[u_{t+1}|s_t\right]$$

Joint surplus is thus given by

$$Q_t=(1-\delta)\left[\underline{y}+f(e_t)\Delta y-c(e_t+s_t)\right]+\delta\mathbb{E}_r\left[Q_{t+1}|s_t\right]$$

Notice that the joint surplus is independent of r_t and under full rent extraction, e_t and s_t are functions of b_t only. Let Q_t^{NR} be the maximum surplus that can be generated when $d_t=1$ and let b_t^{NR} denote the associated bonus payment that implements Q_t^{NR}. Since e_t and s_t are independent of r_t, so is Q_t^{NR}. Under full rent extraction, the principal thus maximizes her payoff by implementing $Q_t=Q_t^{NR}$ and $Q_{t+1}=Q_t^{NR}$ for any r_{t+1} with $d_{t+1}=1$ by offering the bonus rate b_t^{NR} and by adjusting the fixed wage as to implement $u_t=r_t$. Given b^{NR}, this is achieved by a fixed wage of

$$w^{NR}(r_t)=\frac{r_t}{1-\delta}-f(e(b^{NR}))b^{NR}+c(e(b^{NR})+s(b^{NR}))-\frac{\mathbb{E}(r_{t+1})}{1-\delta}.$$

Further, because periods with $d_t=1$ differ only with respect to r_t, $Q_t^{NR}=Q^{NR}$, $(e_t^{NR},s_t^{NR})=(e_t^{NR},s_t^{NR})$. This proves item (*i*). Item (*ii*) directly follows from Lemma 3 and the fact that $u_t=r_t$ for all t.

To prove item (*iii*), notice that stationarity of the joint surplus and (PC) implies $\rho_t=\rho^{NR}=Q^{NR}-\overline{\pi}$. Q^{NR} can thus be written as

$$Q^{NR}=\frac{1-\delta}{1-\delta G(\rho^{NR}|s)}\left[\underline{y}+f(e)\Delta y-c(e+s)\right]+\frac{\delta\left[1-G(\rho^{NR}|s)\right]}{1-\delta G(\rho^{NR}|s)}\left[\overline{\pi}+\mathbb{E}_r\left(r_{t+1}|r_{t+1}>\rho^{NR},s\right)\right],$$

where $(e,s)=(e(b^{NR}),s(b^{NR}))$. The assumptions on the production technology ensure that in deriving the optimal incentive contract, the first-order approach can be applied. Let Q_e and Q_s denote the partial derivatives of a stationary joint surplus with respect to e and s respectively. When $u_t=r_t$ for all t the principal maximizes her payoff by maximizing Q subject to (IC_e) and (IC_s^{NR}). The corresponding first-order condition is given by:

$$Q_e \frac{\partial e^A}{\partial b} + Q_s \frac{\partial s^A}{\partial b} = 0.$$

Lemma 9 implies $Q_s < 0$. Together with (2), this implies that $Q_e < 0$ in the above expression. From (FB_e) and (IC_e) it then follows that we must have $b^{NR} > \Delta y$. □

B.1 Efficiency Wages

Marginal Effects of r_0. When the principal implements the wage floor r_0, the agent's optimal decision (e^A, s^A) is defined by (IC_e) and

$$\frac{\delta}{1-\delta}\left[G_s(r_0|s_t)r_0 + \int_{r_0}^{\overline{r}} r g_s(r|s_t)\right] = c'(e_t + s_t). \qquad (IC_s'')$$

Using the implicit function theorem, we get:

$$\frac{\partial e^A}{\partial r_0} = \frac{-\frac{\delta}{1-\delta}G_s(r_0|s)c''(e+s)}{[f''(e)b - c''(e+s)]\,\frac{\delta}{1-\delta}\int_{\underline{r}}^{\overline{r}} u(r, d_{t+1})g_{ss}(r|s)dr - f''(e)\cdot b\cdot c''(e+s)} > 0,$$

$$\frac{\partial s^A}{\partial r_0} = \frac{-\frac{\delta}{1-\delta}G_s(r_0|s)\,[f''(e)b - c''(e+s)]}{[f''(e)b - c''(e+s)]\,\frac{\delta}{1-\delta}\int_{\underline{r}}^{\overline{r}} u(r, d_{t+1})g_{ss}(r|s)dr - f''(e_t)\cdot b\cdot c''(e+s)} < 0,$$

$$\frac{\partial(e^A + s^A)}{\partial r_0} = \frac{-\frac{\delta}{1-\delta}G_s(r_0|s)f''(e)b}{[f''(e)b - c''(e+s)]\,\frac{\delta}{1-\delta}\int_{\underline{r}}^{\overline{r}} u(r, d_{t+1})g_{ss}(r|s)dr - f''(e)\cdot b\cdot c''(e+s)} < 0.$$

The signs of the respective marginal effects follow from the assumptions on $f(.)$, $c(.)$ and $G(.|.)$. □

Proof of Corollary 1. The result follows immediately from the fact that $\frac{\partial e_t^A + s_t^A}{\partial b_t} > 0 > \frac{\partial e_t^A + s_t^A}{\partial r_0}$. □

Proof of Proposition 1. Fix any $r_0 < \rho^{FB}$ and subtract the left-hand side of (IC_s'') from the left-hand side of (FB_s), giving

$$\frac{\delta G_s(\rho^{FB}|s)}{1-\delta G(\rho^{FB}|s)}\left[T(e,s) - \mathbb{E}_r\left(r|r > \rho^{FB}, s\right) - \overline{\pi}\right] \qquad \text{(B.8)}$$
$$-\frac{\delta}{1-\delta}\left[G_s(r_0|s)r_0 - G_s(\rho^{FB}|s)\mathbb{E}_r\left(r|r > \rho^{FB}, s\right) + \int_{r_0}^{\rho^{FB}} r g_s(r|s)dr\right].$$

Noting that

$$G_s(r_0|s)r_0 + \int_{r_0}^{\rho^{FB}} r g_s(r|s)dr = G(\rho^{FB}|s)\frac{\partial \mathbb{E}\left[u(r,1)|r \le \rho^{FB}, s\right]}{\partial s} + G_s(\rho^{FB}|s)\mathbb{E}\left[u(r,1)|r \le \rho^{FB}, s\right],$$

after some algebra, (B.8) becomes

$$\frac{\delta}{1-\delta}\left\{G_s(\rho^{FB}|s)\left[T(e,s) - \overline{\pi} - \mathbb{E}\left[u(r,1)|r \le \rho^{FB}, s\right]\right] - G(\rho^{FB}|s)\frac{\partial \mathbb{E}\left[u(r,1)|r \le \rho^{FB}, s\right]}{\partial s}\right\}.$$

Because $T(e,s) \ge \overline{\pi} + \mathbb{E}\left[u(r,1)|r \le \rho^{FB}, s\right]$, this expression is unambiguously negative, implying $Q_s < 0$ for any effort choice by the agent that satisfies (IC).

Further, given any $r_0 < \rho^{FB}$, the first-order condition determining $b(r_0)$ is given by:

$$Q_e \frac{\partial e^A}{\partial b} + Q_s \frac{\partial s^A}{\partial b} = 0, \qquad \text{(B.9)}$$

$Q_s < 0$ implies that $Q_e < 0$ in (B.9). From (FB_e) and (IC_e) it then follows that $b(r_0) > \Delta y$ for any $r_0 < \rho^{FB}$.

Furthermore, (B.9) and Corollary 1 imply that at any pair $(b(r_0), r_0)$, with $r_0 < \rho^{FB}$, joint surplus can be raised by increasing r_0. It follows that $Q'(r_0) > 0$. $\square$

Proof of Lemma 5. I first prove that $Q(r_0)$ is concave. Since $f(e)$ and $G(r|s)$ are assumed to be continuously differentiable, $Q(r_0)$ is also continuous and differentiable on $\left[\overline{r}, \rho^{FB}\right]$. Since $r_0 = \rho^{FB}$ maximizes joint surplus, we must have $Q'(\rho^{FB}) = 0$. Further, since $\frac{\partial e^A}{\partial r_0}$ and $|\frac{\partial s^A}{\partial r_0}|$ are finite for any $\left(\underline{r}, \rho^{FB}\right]$, so must be $Q'(r_0)$. It follows that $Q(r_0)$ is concave on $\left[\overline{r}, \rho^{FB}\right]$.

Next, if $Q'(\underline{r}) \geq 1$, then, there is some $\epsilon > 0$ such that (EC) holds for $r_0 = \underline{r} + \epsilon$. If $Q'(\underline{r}) < 1$, then, by concavity of $Q(r_0)$, $Q(r_0) - Q^{NR} < r_0 - \underline{r}$ for all $r_0 \in \left(\underline{r}, \rho^{FB}\right]$, proving that the condition is both necessary and sufficient. $\square$

Proof of Proposition 2. By Lemma 3, implementing first-best effort incentives requires $r_0 = \rho^{FB}$. In that case, the left-hand side of (EC) is equal to $Q^{FB} - \rho^{FB}$, which, by construction of ρ^{FB} is just equal to $\overline{\pi}$. However, because $Q^{NR} - \underline{r} > \overline{\pi}$, the right-hand side of (EC) is always strictly larger than $\overline{\pi}$, which shows that a wage floor $r_0 = \rho^{FB}$ can never be made self-enforcing. $\square$

B.2 Alternative Strategies to Reduce the Returns from Search

In this subsection, I illustrate that the form of the optimal strategy that reduces the agent's returns from search effort is not necessarily given by (4) but instead depends on the details of the function $G(.|.)$.

First, consider a strategy under which the principal pays the agent a rent only for intermediate realizations of $r_t \in [\underline{r}, \overline{r}]$ such that

$$u_t(r,1) = \begin{cases} r, & \text{if } r \leq \underline{r}_0, \\ \overline{r}_0, & \text{if } \underline{r}_0 < r \leq \overline{r}_0, \\ r, & \text{if } r > \overline{r}_0. \end{cases} \tag{B.10}$$

Compared to the wage-floor strategy (4), this strategy has the drawback that it may actually increase the rent-seeking motive, because realizations in the range $[\underline{r}_0, \overline{r}_0]$ are more attractive relative to realizations $r_t < \underline{r}_0$ than when there is no rent payment at all. However, the strategy given by (B.10) may be optimal for particular specifications of $G(.|.)$, e.g. if changes in s do not transfer much mass from the range $[\underline{r}, \underline{r}_0]$ to $[\overline{r}_0, \overline{r}]$ and if they have only a weak effect on the conditional distribution over $[\underline{r}, \underline{r}_0]$. In these cases, paying a wage floor $r_0 < \overline{r}_0$ would not have a strong impact on the agent's search incentives. Further, the agent's rent-seeking motive would not increase much by paying rents in the interval $[\underline{r}_0, \overline{r}_0]$.

Next, consider a step-function that pays different wage floors depending on the range into which r_t falls:

$$u_t(r,1) = \begin{cases} \underline{r}_0, & \text{if } r \leq \underline{r}_0, \\ \overline{r}_0, & \text{if } \underline{r}_0 < r \leq \overline{r}_0, \\ r, & \text{if } r > \overline{r}_0. \end{cases} \tag{B.11}$$

In fact, when $\underline{r}_0 - \underline{r} \geq \overline{r}_0 - \underline{r}_0$, then if the strategy given by (4) with $r_0 = \underline{r}_0$ is enforceable, then so is the strategy given by (B.11). Further, compared to the single-wag-floor strategy (4), (B.11) reduces the agent's job-switching motive, even though it may increase the rent-seeking motive because sampling an outside option in $[\underline{r}_0 < r \leq \overline{r}_0]$ becomes more attractive. Again, (B.11) may be optimal if changes in s do not transfer much mass $[\underline{r}, \underline{r}_0]$ to $[\overline{r}_0, \overline{r}]$.

Finally, consider the strategy that pays r_0 for any $r_t \leq \tilde{r}_0 < r_0$. This strategy reduces the agent's search incentives by rewarding low outside options and in fact making them even more attractive than outside options in the range $[\tilde{r}_0, r_0]$. However, this strategy is more costly than one with $r_0 = \tilde{r}_0$, plus, the principal could further reduce the job-switching motive by raising $\tilde{r}_0$.

The preceding illustrations show that the optimal strategy to reduce the agent's returns from search effort depends on the precise specification of $G(.|.)$. However, the single-wage-floor policy characterized by (4) is the unique strategy that unambiguously reduces the agent's search incentives for any specification of $G(.|.)$ that satisfies the assumptions made in section 3. Furthermore, the first-best incentive contract and the optimal incentive contract under full rent extraction are both special cases of (4).

B.3 The Optimal Incentive Contract

Proof of Proposition 3. To prove item (i), note that for any $r_0 < \rho^{FB}$, players' payoffs on the equilibrium path at any date t are

$$\pi_t = (1-\delta)\left[\underline{y} + f(e_t)(\Delta y - b_t) - w_t\right] + \delta \mathbb{E}_r\left[\pi_{t+1}|r_0, s_t\right]$$

$$u_t = (1-\delta)\left[w_t + f(e_t)b_t - c(e_t + s_t)\right] + \delta \mathbb{E}_r\left[u_{t+1}|r_0 s_t\right]$$

And joint surplus is given by

$$Q_t = (1-\delta)\left[\underline{y} + f(e_t)\Delta y - c(e_t + s_t)\right] + \delta \mathbb{E}_r\left[Q_{t+1}|r_0, s_t\right]$$

Given r_0, the optimal incentive contract maximizes the joint surplus. Let $Q(r_0)$ be the maximum surplus that can be implemented and let $b(r_0)$ be the bonus that implements $Q(r_0)$. On the equilibrium path, Q_t is independent of r_t. Hence, payoff-maximization requires that continuation payoffs are sequentially optimal, i.e. $Q_{t+1} = Q(r_0)$ for any r_{t+1}. It follows that for any r_0, the optimal bonus $b(r_0)$ is stationary.

Further, (EC) implies that if there exist efficiency wages that are self-enforcing, the principal earns a strictly higher payoff in the corresponding PPE than in the PPE without efficiency wages. Item (ii) then follows from Lemma 5.

Moreover, Proposition 2 implies that $r_0^* < \rho^{FB}$. Item (v) then follows from Lemma 3 and item (ii) follows from Proposition 1. Further, since $Q_e < 0$ and $Q_s < 0$, (FB_e) and (FB_s) imply $e^* + s^* > e^{FB} + s^{FB}$.

Finally, to see which efficiency wage $r_0 \in [\underline{r}, r_0^{max}]$ the principal prefers to implement, note that the contracting parties must agree on r_0 $t = 1$. The optimal efficiency wage, r_0^* then maximizes the principal's supergame payoff from the perspective of $t = 1$, which is $Q(r_0) - u_1(r_1, 1)$. Since there is no previous period in which the agent could influence his date-1 outside option, PPE does not require that the wage floor be paid in the initial period. Thus, the principal can promise to pay $r_0 \in (\underline{r}, r_0^{max}]$ from $t = 2$ onward, thereby implementing $Q(r_0)$ and extract the full rent in period $t = 1$. Since $Q'(r_0) > 1$, the optimal efficiency wage is thus given by r_0^{max}. □

Proof of Proposition 4. Suppose $s^* = 0$ under the optimal incentive contract. Since for any $r_0 < \rho^{FB}$, $Q_s < 0$, $s^* = 0$ implies $s^{FB} = 0$. Conditional on $s^* = 0$, the principal implements a level of work effort that maximizes the joint surplus, i.e. $Q_e = 0$, which requires $b^* = \Delta y$. In turn, (IC_e) implies that for $s^* = 0$ and $b^* = \Delta y$, the agent chooses $e^* = e^{FB}$. But by (IC_e) and (IC_s), this is consistent with $s^* = 0$ only if

$$f'(e^{FB})\Delta y = c'(e^{FB}) > \frac{\delta}{1-\delta} \frac{\partial E_r\left(r_{t+1}|s=0\right)}{\partial s_t}.$$

To prove sufficiency, suppose that (6) holds. In that case, offering $b^* = \Delta y$ and $r_0^* = \overline{r}$ induces the agent to choose $(e^*, s^*) = (e^{FB}, 0)$. □

C Renegotiation

Proof of Lemma 6. Consider a PPE that pays r_0 only from some date $t > 1$ onward. Then, at date $t-1$, the agent will choose (e_{t-1}, s_{t-1}) expecting to receive r_0 from period t onward, generating a joint surplus of $Q(r_0)$. Further, suppose that $r_t < r_0$ but that at date t, the principal offers the agent a spot contract that would give him a payoff of r_t if he chose the same effort profile as in period $t-1$. If the agent accepts the contract and subsequently reverts to the full-rent-extraction PPE he receives at most r_t and generates a joint surplus of Q^{NR}. If he rejects it, he receives r_t and the joint surplus is $\overline{\pi}+r_t$. Further, if he accepts the contract and subsequently chooses the same effort profile as in $t-1$, he also receives r_t but generates a joint surplus of $Q(r_0) > Q^{NR} > \overline{\pi} + r_t$. Thus, the agent is indifferent between reverting to the continuation equilibrium from $t-1$ and carrying out the punishment prescribed by the PPE. By contrast, under punishment, the principal receives a payoff $\leq Q^{NR} - r_t$ which is strictly less than, $Q(r_0) - r_t$, which she gets when reverting to the continuation equilibrium from period $t-1$. Hence, switching to the continuation equilibrium that started at $t-1$ represents a Pareto-improvement implying that the PPE does not satisfy WRP.

Conversely, consider a PPE under which r_0 is paid from date $t = 1$ on. Further, consider some date t with $r_t < r_0$ at which the principal makes an out-of-equilibrium offer that yields the agent at most r_t. The original PPE has no continuation equilibrium under which the agent accepts such an offer and subsequently chooses (e_t, s_t) as if expecting to receive r_0 from period $t+1$ onward. Hence, such a PPE satisfies WRP. □

Proof of Lemma 7. By Lemma 6, the optimal incentive contract must pay r_0 from date $t = 1$ on. Thus, when choosing the efficiency wage in period 1, the principal's problem is

$$\max_{r_0} \left[Q(r_0) - \max(r_1, r_0)\right] \; \textit{subject to} \; (EC) \tag{C.1}$$

Since $\widehat{r}_0$ maximizes $Q(r_0) - r_0$, the solution to this problem is $\widehat{r}_0$ for any $r_1 \leq \widehat{r}_0$. By contrast, when $r_1 \geq \widehat{r}_0$, it is not profitable to raise r_0^* above r_1. Hence, in that case, $r_0^* = r_1$ as long as $r_1 \leq r_0^{max}$. If $r_1 \geq r_0^{max}$, (EC) implies $r_0^* = r_0^{max}$.

For $r_0 > \widehat{r}_0$, concavity of $Q(r_0)$ implies $Q'(r_0) - 1 < 0$. But by (PC), $r_1 \geq r_0$. It follows that $r_0 = r_1$, if $r_0 > \widehat{r}_0$ and $r_1 \leq r_0^{max}$. If $r_1 > r_0^{max}$, the principal maximizes her payoff by implementing the highest payoff possible, hence, $r_0 = r_0^{max}$. □

Proof of Proposition 5. Consider a repeated game with $r_1 < r_0^{max}$. Further, define $r_0(t)$ as the efficiency wage that is in place in period t. Lemma 7 implies that at any date $\tau > t$ with $r_0(t) < r_\tau \leq r_0^{max}$, the principal can raise her payoff by changing r_0^* to r_τ. Further, because such a change also raises the agent's payoff, he would accept such a change. Thus, if players can renegotiate r_0, we have that for any $\tau > t$, $r_0(\tau) \geq r_0(t)$, with strict inequality if $r_0(t) < r_{\widetilde{t}} \leq r_0^{max}$ for some $\widetilde{t}$ with $t < \widetilde{t} \leq \tau$. Without renegotiation, the agent's expected payoff at date t is $E_r(u_t) = G(r_0|s_{t-1})r_0 + \int_{r_0}^{\overline{r}} rg(r|s_t)dr$. With renegotiation, the agent's expected payoff depends on last period's efficiency wage:

$$E_r(u_t|r_0(t-1)) = E_r\left[G(r_0|s_{t-1})r_0 + \int_{r_0}^{\overline{r}} rg(r|s_t)dr\right],$$

Since the efficiency wage will only be renegotiated upward, this expression is increasing in $r_0(t-1)$ which proves item (i).

Item (ii) readily follows from Proposition 1 and the fact that r_0 is (weakly) increasing over time.

To prove item (iii), note that the separation rate at date t is given by $1 - G\left[\rho(r_0(t))|s((r_0(t))\right]$. Since joint surplus increases over time, so does the separation threshold $\rho(r_0(t))$, which reduces the separation rate since $G_r(.|.)$. Also, the agent's optimal level of search effort decreases with the efficiency wage which, because $G_s < 0$, reduces the separation rate as well. □

D Extensions

Proof of Lemma 8. Adding b^{max} and $-b^{min}$ gives:

$$\Delta b^{max} \equiv \frac{\delta}{1-\delta} G(\rho|s)\left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) - \overline{\pi}\right].$$

Differentiating this expression with respect to r_0 yields:

$$\begin{aligned}\frac{\partial \Delta b^{max}}{\partial r_0} &= \delta g(\rho|s)\left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) - \overline{\pi}\right]\frac{\partial \rho}{\partial Q}\frac{\partial Q}{\partial r_0} \\ &+ \delta G_s(\rho|s)\left[Q(e,s) - \mathbb{E}\left(r|r \leq \rho, s\right) - \overline{\pi}\right]\frac{\partial s}{\partial r_0} \\ &+ \delta G(\rho|s)\left[\frac{\partial Q}{\partial r_0} - \frac{\partial \mathbb{E}\left(r|r \leq \rho, s\right)}{\partial s}\frac{\partial s}{\partial r_0} - \frac{\partial \mathbb{E}\left(r|r \leq \rho, s\right)}{\partial \rho}\frac{\partial \rho}{\partial Q}\frac{\partial Q}{\partial r_0}\right].\end{aligned}$$

Using

$$\frac{\partial \mathbb{E}\left(r|r \leq \rho, s\right)}{\partial \rho} = \frac{g(\rho|s)}{G(\rho|s)}\left[\rho - \mathbb{E}\left(r|r \leq \rho, s\right)\right],$$

$$\frac{\partial \rho}{\partial Q} = \frac{1}{1 - \delta G(\rho|s)},$$

and canceling terms gives

$$\begin{aligned}\frac{\partial \Delta b^{max}}{\partial r_0} &= \delta G_s(\rho|s)\left[Q(e,s) - \mathbb{E}(r|r \leq \rho, s) - \overline{\pi}\right]\frac{\partial s}{\partial r_0} \\ &+ \delta G(\rho|s)\left[\frac{\partial Q}{\partial r_0} - \frac{\partial \mathbb{E}\left(r|r \leq \rho, s\right)}{\partial s}\frac{\partial s}{\partial r_0}\right].\end{aligned} \tag{D.1}$$

Since $Q(e,s) > \mathbb{E}\left(r|r \leq \rho, s\right) + \overline{\pi}$. Further,

$$\frac{\partial \mathbb{E}\left(r|r \leq \rho, s\right)}{\partial s} > 0,$$

It follows that

$$\frac{\partial \Delta b^{max}}{\partial r_0} > 0.$$

□

Proof of Proposition 6. If (DE) is slack, $\Delta\widetilde{b}^* = b^*$ and $\widetilde{r}_0^* = r_0^*$. By contrast, if it binds, $\Delta\widetilde{b}^* < b^*$. Further, by Lemma 8, the marginal increase in surplus caused by raising r_0 is strictly larger than when b is enforceable. However, this can only lead to $\widetilde{r}_0^* = r_0^*$ when efficiency wages are self-enforcing, i.e. if (EW) holds. □

The Agent's Incentive Compatibility Constraints under Limited Liability. When the principal offers a stationary contract, the agent's payoff under limited liability at every date t with $d_t = 1$ is

$$\rho = \frac{1-\delta}{1-\delta G(\rho|s)}\left[f(e)b - c(e+s)\right] + \frac{\delta}{1-\delta G(\rho|s)}\int_{\rho}^{\overline{r}} rg(r|s)dr. \tag{D.2}$$

Further, define

$$H(e,s;\rho,b) = \frac{1-\delta}{1-\delta G(\rho|s)}\left[f(e)b - c(e+s)\right] + \frac{\delta}{1-\delta G(\rho'|s)}\int_{\rho}^{\overline{r}} rg(r|s)dr - \rho = 0.$$

Differentiating this expression with respect to ρ gives:

$$\frac{\partial H(e,s;\rho,b)}{\partial \rho} = \frac{\delta g(\rho|s)}{1-\delta G(\rho|s)}\rho - \frac{\delta g(\rho|s)}{1-\delta G(\rho|s)}\rho - 1 = -1$$

Hence, by the implicit function theorem, the derivative of ρ with respect to e and s is just equal to the derivative of the right-hand side of (D.2). Therefore, the first-order conditions of the agent's problem under limited liability are identical to (IC_e) and (IC_s'') with $r_0 = \rho$.

Differentiating (D.2) with respect to b gives:

$$\frac{d\rho}{db} = \frac{\partial \rho}{\partial e}\frac{\partial e}{\partial b} + \frac{\partial \rho}{\partial s}\frac{\partial s}{\partial b} + \frac{\partial \rho}{\partial b}.$$

By the envelope theorem, the first two terms of this expression are equal to zero, hence:

$$\frac{\partial \rho}{\partial b} = \frac{\partial \rho}{\partial b} = \frac{1-\delta}{1-\delta G(\rho|s)} f(e) > 0.$$

□

Proof of Proposition 7. The proof proceeds by showing that there is no r_t for which the principal could raise her profits by deviating to a contract offer different from the one specified in Proposition 7. b_0^{ll} represents the bonus that maximizes the principal's payoff conditional on the continuation play displayed in the Proposition. Given this continuation play, the agent's payoff when $b = b_0^{ll}$ is r_0^{ll}. Thus, whenever $r_t \leq r_0^{ll}$, the agent will accept the contract $b = b_0^{ll}$ and, by construction, there is no other bonus that yields the principal higher profits. Further, since (IC_e) and (IC_s'') have a unique solution, the agent chooses the same effort profile (e_0^{ll}, s_0^{ll}) whenever $b = b_0^{ll}$. It follows that the principal's profit and the agent's payoff will be the same whenever $r_t \leq r_0^{ll}$.

When $r_t \in \left(r_0^{ll}, \rho^{ll}\right]$, the principal must offer a contract that yields the agent a higher payoff than r_0^{ll}. This can either be done by raising the fixed wage w_t or by offering a higher bonus payment. For $r > r_0^{ll}$ and given $b = b_0^{ll}$, the fixed wage that the principal must offer to satisfy the agent's participation constraint is

$$w(r) = \frac{1-\delta G(\rho^{ll}|s)}{1-\delta}\left(r - r_0^{ll}\right) > r - r_0^{ll}.$$

Since $w(r)$ is a mere transfer, it reduces the principal's profits by exactly $w(r)$. Alternatively, the principal can raise the bonus to meet the agent's participation constraint. Let $b^{ll}(r) > b_0^{ll}$ denote the bonus payment such that $u = r$. Under limited liability, the principal will offer a contract only if $b^{ll} < \Delta y$. However, since total surplus is increasing in b and r_0 for all $b \leq \Delta y$, raising b above b_0^{ll} increases total surplus. Let $\Delta Q(r) > 0$ denote the associated increase in total surplus. The principal's costs of raising b from b_0^{ll} to $b^{ll}(r)$ are hence equal to $(r - r_0^{ll}) - \Delta Q(r) > 0$, which is strictly smaller than $(r - r_0^{ll})$. It follows that the principal can raise the agent's payoff more cheaply by increasing b instead of w.

Further, by construction, any increase of b above b_0^{ll} reduces the principal's profits. Thus, the contract that maximizes the principal's profits for any $r_t > r_0^{ll}$ is $b^{ll}(r_t)$. Since $b^{ll}(r)$ is increasing in r, there is some $r_t \rho^{ll}$ such that $\pi^{ll}(\rho^{ll}) = \overline{\pi}$ and for all $r > \rho^{ll}$ the profit-maximizing contract offer that satisfies the agent's participation constraint yields the principal profits smaller than $\overline{\pi}$. Hence, ρ^{ll} constitutes the equilibrium separation threshold. Finally, by (2), e^{ll} (s^{ll}) is increasing (decreasing) in $b^{ll}(r)$.

□

Proof of Proposition 8. The result that the optimal incentive contract is stationary and inefficient is independent of the precise specification of the cost function. Thus, what remains to be shown is that the optimal incentive contract features excessive bonuses and an efficiency wage. Note that under the general cost function $c(e, s)$, the first-order conditions of the agent's problem are given by:

$$f'(e_t)b_t = c_e(e_t, s_t),$$

$$\frac{\delta}{1-\delta}\left[G_s(r_0|s_t)r_0 + \int_{r_0}^{\overline{r}} r g_s(r|s_t)\right] = c_s(e_t, s_t).$$

Using the implicit function theorem, we get:

$$\frac{\partial s_t^A}{\partial b} = \frac{-f'(e_t)c_{es}}{[f''(e_t)b - c_{ee}]\left\{-\frac{\delta}{1-\delta}\left[G_{ss}(r_0|s_t)r_0 + \int_{r_0}^{\overline{r}} r g_{ss}(r|s_t)dr\right] - c_{ss}\right\}[f''(e_t)\cdot b_t - c_{ee}] + c_{es}^2}.$$

When $c_{es} > 0$, this expression is unambiguously positive, thus justifying the use of excessive bonuses. Furthermore, we have:

$$\frac{\frac{\partial e_t^A/\partial b_t}{\partial s_t^A/\partial b_t}}{\frac{\partial e_t^A/\partial r_0}{\partial s_t^A/\partial r_0}} = \frac{\frac{\delta}{1-\delta}\left\{\left[G_{ss}(r_0|s_t)r_0 + \int_{r_0}^{\overline{r}} r g_{ss}(r|s_t)dr\right] - c_{ss}\right\}\left[f''(e_t)\cdot b_t - c_{ee}\right]}{c_{es}^2} \tag{D.3}$$

The second-order sufficient conditions ensuring that (e_t^A, s_t^A) represent the unique maximum of the agent's problem imply that (D.3) is strictly larger than one. Thus, Corollary 1 holds as well, which implies that at the margin, raising the efficiency wage always raises the joint surplus. □

Proof of Proposition 9. When the principal extracts the all gains from trade, PC implies that an employment relationship can be sustained if and only if $Q^{NR} - \underline{r} > \overline{\pi}$. Further, under A2, first-best effort incentives require $r_0 = \overline{r}$. By (EC), $r_0 = \overline{r}$ is self-enforcing if and only if $Q^{FB} - \overline{r} \geq Q^{NR} - \underline{r} > \overline{\pi}$. □

Proof of Proposition 10. Under long-term contracting, the principal's problem amounts to choosing the sequence $\{(w_t, b_t)\}_{t=1}^{\infty}$ that maximizes his payoff at date $t = 1$, $Q_1 - u_1$ subject to (PC), (IC_e) and (IC_s). Conditional on r_1, the highest value that the principal's payoff can take on is $Q^{FB} - r_1$. Further, under long-term contracting, the agent's effort incentives at date t are independent of the rent he receives in t, because there is no time-inconsistency problem on behalf of the principal. Hence, the principal sets $u_1 = r_1$ and implements Q^{FB} by setting $b_t = \Delta y$ for all t and $u_t = \rho^{FB}$ for all $t \geq 2$ with $d_t = 1$ (see Lemma 3). □

E Endogenizing the Set of Outside Options

Recall that a draw of the outside option $r \in [\underline{r}, \overline{r}]$ represents the value of a job in the supergame rather than the maximum wage the employer providing job r could offer. In this section, I delineate how the interval $[\underline{r}, \overline{r}]$ could be endogenized within a model of a labor market with a continuum of jobs, in which the agent can potentially work and that differ with respect to the agent's productivity in each job.

Suppose that there is a mass J of jobs in which the agent can potentially work. Jobs are indexed by $j \in [0, J]$. Every job j is associated with a principal and characterized by a moral hazard problem of the type described in section 3. Thus, the agent chooses work and search effort in every period, irrespective of which job he is currently working in. However, jobs or principals differ with respect to their productivity. Let $Q(j; \underline{r}, \overline{r})$ denote the equilibrium surplus that the agent generates in job j under the optimal incentive contract given arbitrary values for $\underline{r}$ and $\overline{r}$. That is, $Q(j; \underline{r}, \overline{r})$ represents the surplus created under the optimal incentive contract in job j. I assume that $Q_j(j; \underline{r}, \overline{r}) > 0$, i.e. jobs with a higher productivity are represented by higher indexes. Differences in productivity could for instance be generated by differences in the production technology, i.e. jobs could differ with respect to $f_i(e)$, $\underline{y}_i$ or Δy_i. Principal j's maximum willingness-to-pay for the agent's services is therefore given by $\omega_j = Q(j, \underline{r}, \overline{r}) - \overline{\pi}$. Thus, every range of outside options $[\underline{r}, \overline{r}]$ generates a range $[\underline{\omega}, \overline{\omega}]$, where $\underline{\omega}$ and $\overline{\omega}$ denote the lowest and highest willingness-to-pay for the agent's services over all firms.

Let j_t denote the firm that employed the agent at date $t-1$ and suppose that at the start of period t, the agent samples N_t jobs from the population $[0, J]$ such that he entertains $N_t + 1$ job offers at the start of date t. Let Ω_t denote the set of willingnesses to pay of all new firms sampled at date t and define $\omega_t^{max} \equiv \max\{\omega_j\}_{\omega_j \in \Omega_t}$. Assuming that firms engage in Bertrand competition for the agent, his outside option when staying with firm j_t (i.e. if $\omega_{j_t} \geq \omega_t^{max}$) is ω_t^{max}. By contrast, if he switches firms (i.e. if $\omega_{j_t} < \omega_t^{max}$), his continuation payoff is bounded by ω_t^{max}. In particular, even though firms engage in Bertrand competition, the agent's continuation payoff may well be larger than his second-highest offer, because his new employer might prefer to pay him an efficiency wage. Thus, the range of the agent's outside options is endogenously determined by firms' willingnesses to pay for the agent's services, $[\underline{\omega}, \overline{\omega}]$ However, recall that this range depends on the specification of $\underline{r}$ and $\overline{r}$. Thus in equilibrium $[\underline{\omega}, \overline{\omega}] = [\underline{r}, \overline{r}]$. In such an equilibrium, each firm takes the range $[\underline{r}, \overline{r}]$ as given, even though it is endogenously determined by each job's equilibrium outcome.

Further, to endogenize the effect of s on $G(.|.)$, specified in section 3, assume that in the sketched model, the expected number of jobs sampled at t conditional on s_{t-1}, $\mathbb{E}(N_t|s_{t-1})$ is increasing in s_{t-1}. In that case, the implied distribution over the agent's outside option at date t is equivalent to the formulation from section 3, because a higher level of search raises $\mathbb{E}(N_t|s_{t-1})$ and therefore also he expected value of the highest outside option.

The model sketched in this section also indicates why it is not admissible to conduct comparative statics on δ in the partial equilibrium model analyzed in the main part of the paper. When the agent's discount factor is the same in every job, changes in δ also affect $[\underline{r}, \overline{r}]$. Therefore, the effects of changes in δ on optimal incentive contracts under OJS can only be assessed in an equilibrium model of the entire labor market.

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